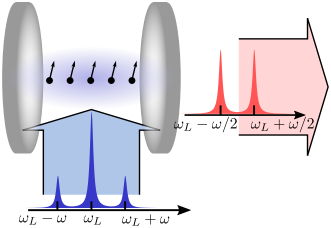
Atom-only descriptions of atom-cavity systems

Simon B. Jäger

30.10.2024

Time-periodically driven atom-cavity setups



Cavity photons mediate time-periodic interactions between the atoms

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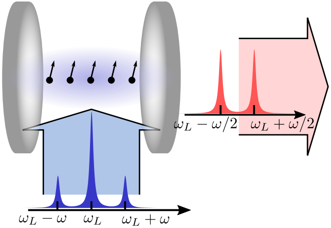

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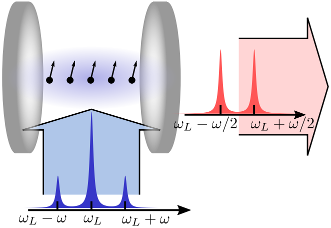

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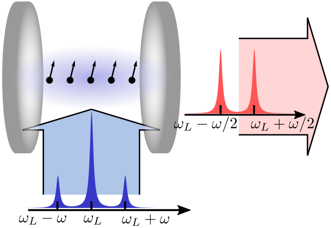
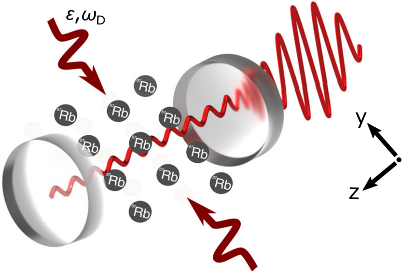




H. Keßler et al., PRL **127**, 043602 (2021).

Cavity photons mediate time-periodic interactions between the atoms

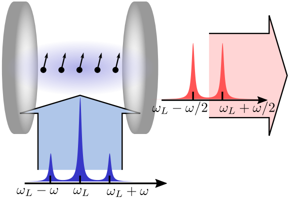
→ resonant creation of coherent spatio-temporal pattern

→ dissipation of cavity photons through mirrors stabilizes pattern

Generation of new phases of matter stabilized by dissipation

→ dissipative time crystals

Effective theoretical description



We are interested in the behavior of the atoms!

Can we eliminate the field degrees of fredom?

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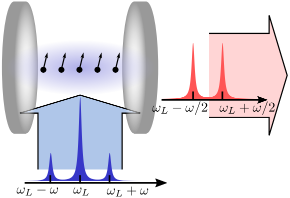

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Gain:
- more efficient description of the dynamics
- analytical results and predictions

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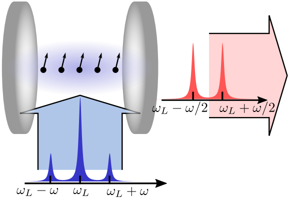

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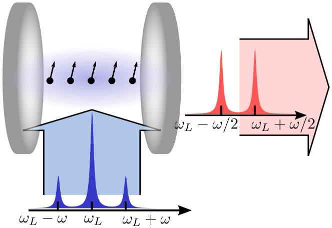
Challenge: correct descriptions of interactions + dissipation!

Periodically driven Dissipative Dicke model



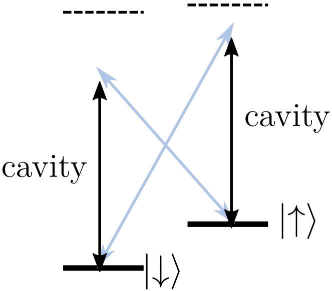
Single diss. cavity mode coupled to collective spin

$$\frac{\partial \hat{\rho}}{\partial t} = -i\left[\hat{H}, \hat{\rho}\right] - \kappa(\hat{a}^\dagger \hat{a}\hat{\rho} + \hat{\rho}\hat{a}^\dagger \hat{a} - 2\hat{a}\hat{\rho}\hat{a}^\dagger)$$

$$\hat{H} = \omega_0 \hat{Z} + \omega_c \hat{a}^\dagger \hat{a} + \frac{2g(t)}{\sqrt{N}}(\hat{a} + \hat{a}^\dagger)\hat{X}$$

$$\hat{X} = \frac{1}{2}\sum_j \hat{\sigma}_j^x,\ \hat{Y} = \frac{1}{2}\sum_j \hat{\sigma}_j^y,\ \hat{Z} = \frac{1}{2}\sum_j \hat{\sigma}_j^z$$

$$g(t) = g_0 + g_1 \cos(\omega t)$$



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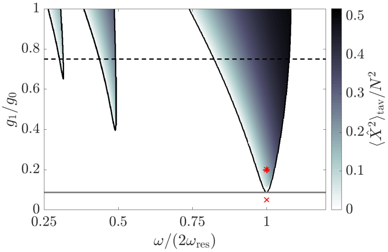

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Exhibits time crystalline phase for parameteric driving

R. Chitra and O. Zilberberg, Phys. Rev. A 92, 023815 (2015).
Z. Gong, R. Hamazaki, and M. Ueda, Phys. Rev. Lett. **120**, 040404 (2018).

- parameteric resonances $\frac{n\omega}{2} = \omega_{\rm res} = \omega_0\sqrt{1 - \frac{4g_0^2\omega_c}{\omega_0[\omega_c^2+\kappa^2]}}$

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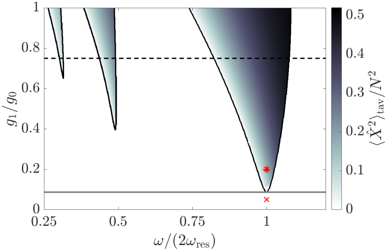

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- signaled by (i) $\langle[\hat{X}]^2\rangle \propto N^2$ and (ii) $\langle \hat{X}(t+t_0)\hat{X}(t_0)\rangle$ is $4\pi/\omega$ periodic in t

$\rightarrow$ Toy model for dissipative time crystals

We want to eliminate the cavity!

S. B. Jäger et al., Phys. Rev. Lett **129**, 063601 (2022).

Derivation of the master equation

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Step 1: Move into frame which decouples cavity and spin $\|\hat{\alpha}\| \ll 1$

$$\tilde{\rho} = \hat{D}^\dagger \hat{\rho} \hat{D}, \; \hat{D} = \exp\left[\hat{\alpha}^\dagger \hat{a} - \hat{a}^\dagger \hat{\alpha}\right]$$

and find operator $\hat{\alpha}$ that minimizes coupling between cavity and spin

$$\rightarrow \frac{\partial \hat{\alpha}}{\partial t} = -i[\omega_0 \hat{Z}, \hat{\alpha}] - (i\omega_c + \kappa)\hat{\alpha} - i\frac{2g(t)}{\sqrt{N}}\hat{X}$$

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Step 2: Project onto the vacuum in the displaced picture $\hat{\rho}_{\text{spin}} = \langle \text{vac}|\tilde{\rho}|\text{vac}\rangle$

$$\frac{\partial \hat{\rho}_{\text{spin}}}{\partial t} = -i\left[\hat{H}_{\text{eff}}, \hat{\rho}_{\text{spin}}\right] - \kappa(\hat{\alpha}^\dagger \hat{\alpha}\hat{\rho}_{\text{spin}} + \hat{\rho}_{\text{spin}}\hat{\alpha}^\dagger\hat{\alpha} - 2\hat{\alpha}\hat{\rho}_{\text{spin}}\hat{\alpha}^\dagger)$$

$$\hat{H}_{\text{eff}} = \omega_0 \hat{Z} + \frac{g}{\sqrt{N}}(\hat{\alpha}^\dagger \hat{X} + \hat{X}\hat{\alpha})$$

Master equation with cavity-mediated Interactions and Dissipation

When is it possible to eliminate the field?

What are the limitations?

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Valid if there is a timescale separation

$$\underbrace{\kappa^{-1}, \omega_c^{-1}}_{\text{timescale of cavity}} \ll \underbrace{\omega_0^{-1}, \omega^{-1}, g^{-1}}_{\text{timescale of atom and drive}}$$

$\rightarrow$ **Cavity adiabatically follows atomic degrees**

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Solution for $\hat{\alpha}$:

$$\hat{\alpha} = \frac{\alpha_+(t)}{\sqrt{N}}\hat{S}^+ + \frac{\alpha_-(t)}{\sqrt{N}}\hat{S}^- \qquad \alpha_\pm \approx -\underbrace{\frac{g(t)}{\omega_c - i\kappa}}_{\omega_c^{-1} g} - i\underbrace{\frac{\dot{g}(t)}{[\omega_c - i\kappa]^2}}_{\omega_c^{-2} g\omega} \pm \underbrace{\frac{\omega_0 g(t)}{[\omega_c - i\kappa]^2}}_{\omega_c^{-2} g\omega_0}$$

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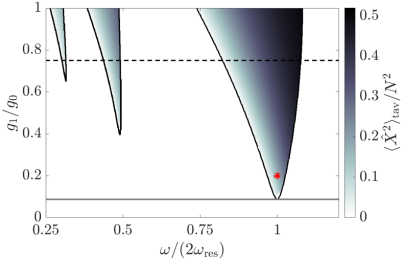
Comparison of atom-cavity and atom-only



Comparison in time-crystalline phase

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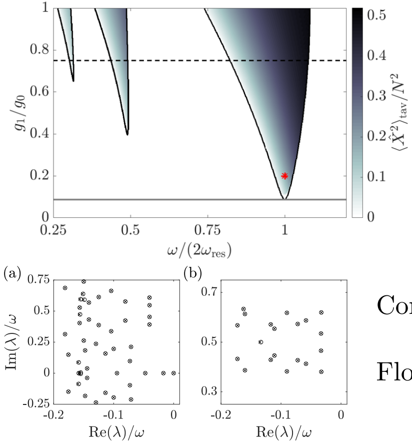

Comparison in time-crystalline phase

Comparison of quantum dynamics ($N \leq 20$)

Floquet spectrum $\lambda\hat{\varrho} = \left[\mathcal{L} - \frac{\partial}{\partial t}\right]\hat{\varrho}$

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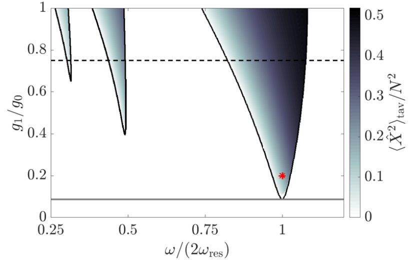

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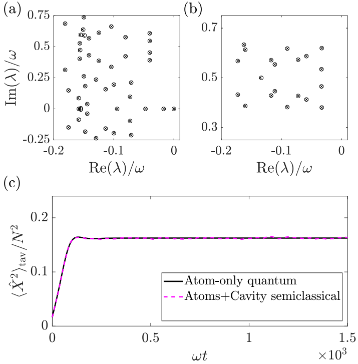

Comparison of quantum dynamics ($N \leq 20$)

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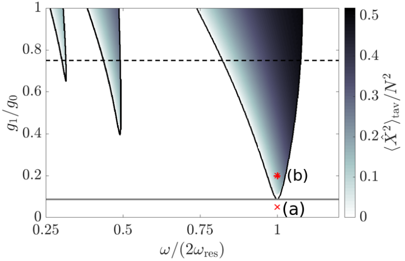
Comparison to semiclassical dyn. ($N \sim 100$)

Excellent agreement!

Spectral features of the time crystalline phase



We are now able to study spectral features for large N

Time-crystal breaks discrete time-translational symmetry

$\rightarrow$ Closing gap at $\lambda = i\frac{\omega}{2} + \gamma$ (subharmonic response)

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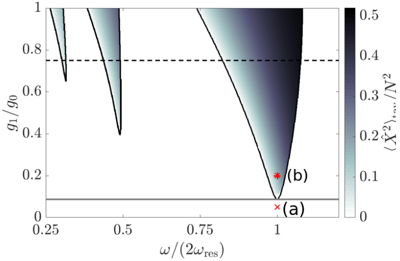

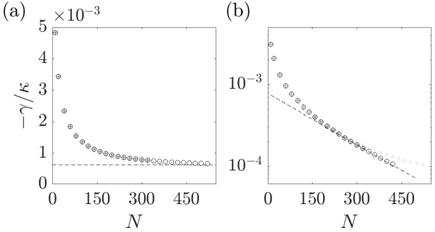

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Time-crystal breaks discrete time-translational symmetry

$\rightarrow$ Closing gap at $\lambda = i\frac{\omega}{2} + \gamma$ (subharmonic response)

(a) λ remains gapped. (b) λ closes exponentially in N

First description of this effect with only atomic degrees of freedom

Atom-only mean field theory

Mean-field description of dynamics

$$\hat{X} = \hat{b}_\uparrow^\dagger \hat{b}_\downarrow + \hat{b}_\downarrow^\dagger \hat{b}_\uparrow,\ \hat{Y} = i(\hat{b}_\uparrow^\dagger \hat{b}_\downarrow - \hat{b}_\downarrow^\dagger \hat{b}_\uparrow),\ \hat{Z} = \hat{b}_\uparrow^\dagger \hat{b}_\uparrow - \hat{b}_\downarrow^\dagger \hat{b}_\downarrow\ ,\ \varphi_s = \langle \hat{b}_s \rangle$$

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$$\frac{d\varphi_\downarrow}{dt} = i\frac{V_0(t) - iV_1(t)}{N}|\varphi_\uparrow|^2\varphi_\downarrow + i\frac{V_0(t) + iV_1(t)}{N}\varphi_\uparrow^2\varphi_\downarrow^*$$

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Time-dependent interactions V_0

and dissipation $V_1 \propto \frac{1}{(\omega-\omega_0)^2+\kappa^2} - \frac{1}{(\omega+\omega_0)^2+\kappa^2}$ (Cooling)

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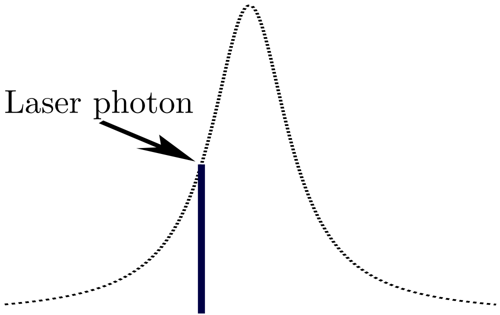

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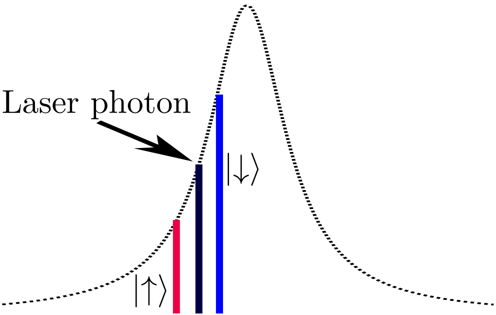

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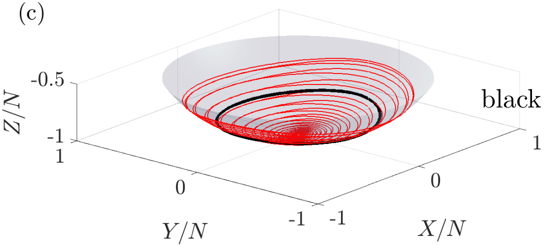

black $\rightarrow$ $V_1 \neq 0$, red $\rightarrow$ $V_1 = 0$

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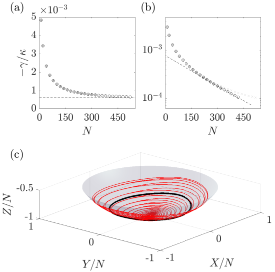
Conclusion



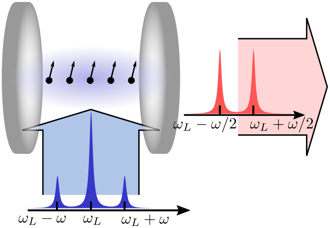
- Description for time-periodic dissipative Dicke model

  → Description of exponentially closing gap

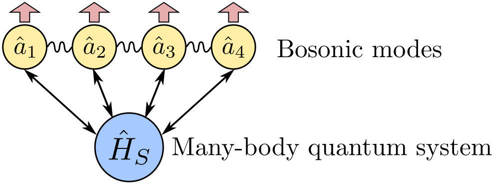
  → New insights into stabilization mechanism

  S. B. Jäger et al., Phys. Rev. A 110, L010202 (2024).

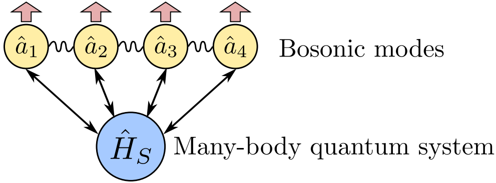
Atom-only description of dissipative Dicke timecrystals



Interaction and Dissipation engineering with bosonic modes



General setup



Quantum system coupled to dissipative bosonic modes $\hat{a}_k$

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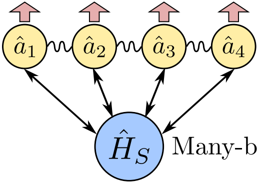
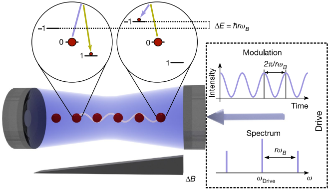
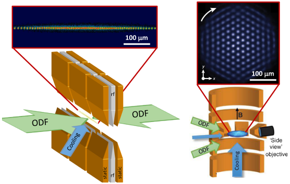


Quantum system coupled to dissipative bosonic modes $\hat{a}_k$

$\rightarrow$ Engineer interactions within the quantum system



A. Periwal et al., Nature **600**, 630 (2021).



C. Monroe et al., RMP **93**, 025001 (2021).

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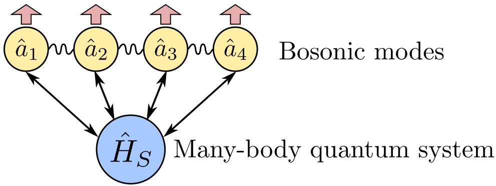

Quantum system coupled to dissipative bosonic modes $\hat{a}_k$

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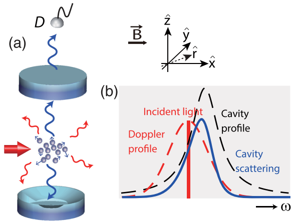
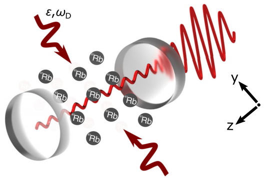
$\rightarrow$ Tailor dissipation for the quantum system

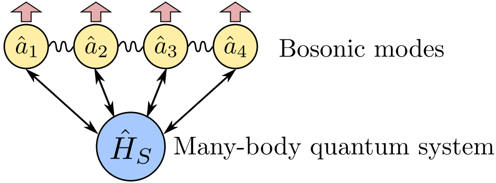


M. Hosseini et al., PRL **118**, 183601 (2017).



H. Keßler et al., PRL **127**, 043602 (2021).

Derivation of the master equation



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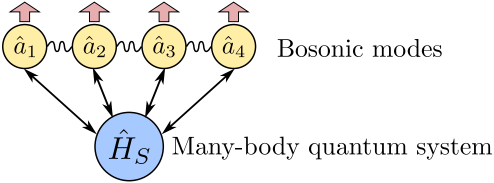

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Move into frame which weakly correlates modes and quantum system $\|\hat{\alpha}_k\| \ll 1$

$$\tilde{\rho} = \hat{D}^\dagger \hat{\rho} \hat{D},\ \hat{D} = \exp\left[\sum_k (\hat{\alpha}_k^\dagger \hat{a}_k - \hat{a}_k^\dagger \hat{\alpha}_k)\right]$$

Find system operator $\hat{\alpha}_k$ that minimizes coupling between modes and system

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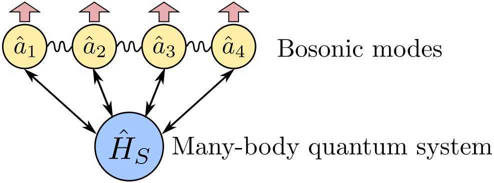

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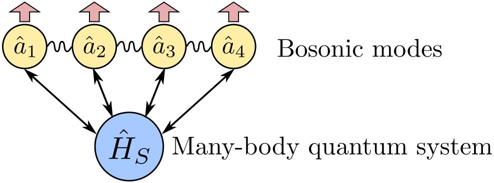
Effective Lindblad master equation



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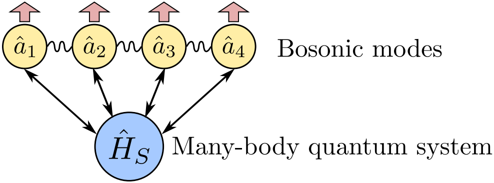

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$$\frac{\partial \hat{\rho}_{\mathrm{sys}}}{\partial t} = -i\left[\hat{H}_{\mathrm{eff}}, \hat{\rho}_{\mathrm{sys}}\right] - \sum_k \kappa_k (\hat{\alpha}_k^\dagger \hat{\alpha}_k \hat{\rho}_{\mathrm{sys}} + \hat{\rho}_{\mathrm{sys}} \hat{\alpha}_k^\dagger \hat{\alpha}_k - 2\hat{\alpha}_k \hat{\rho}_{\mathrm{sys}} \hat{\alpha}_k^\dagger)$$

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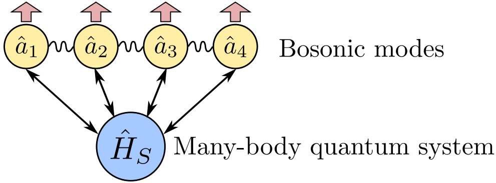

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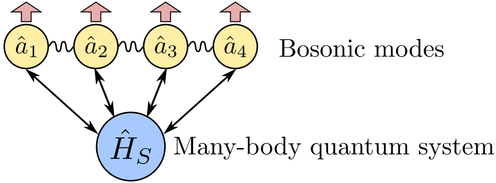
Master equation of Lindblad form → completely positive

Effective Lindblad master equation



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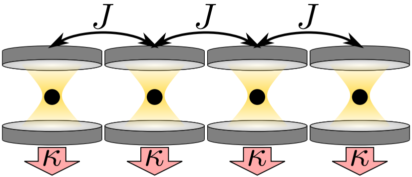
Master equation of Lindblad form $\rightarrow$ completely positive

We assumed $\|\hat{\alpha}_k\| \ll 1 \rightarrow$ "weak" coupling regime

Example 1: Coupled cavities to spins



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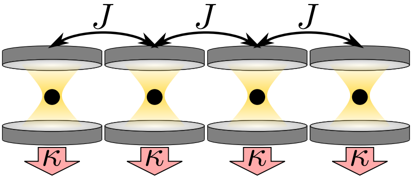

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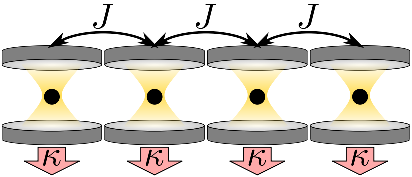

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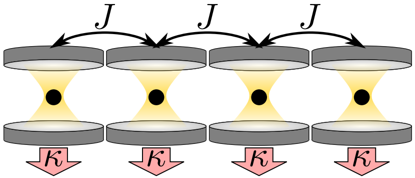

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Solution using Fourier transformation:

$$\hat{\alpha}_m = \frac{2g}{J}\sum_n \left[\frac{|\epsilon_+|^{|n-m|}}{\epsilon_+ - \epsilon_+^{-1}}\hat{\sigma}_n^+ + \frac{|\epsilon_-|^{|n-m|}}{\epsilon_- - \epsilon_-^{-1}}\hat{\sigma}_n^-\right]$$

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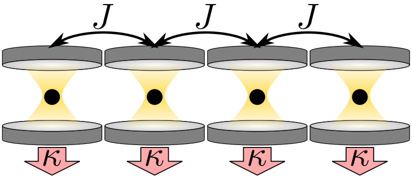

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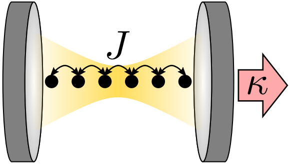

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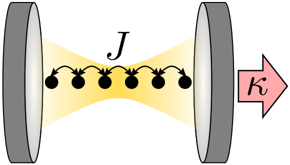

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Can we cool the ising chain to its ground state?

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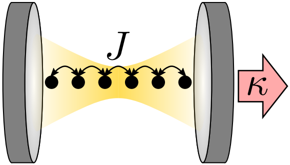

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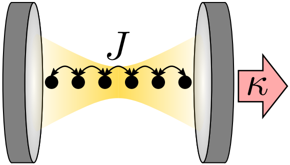

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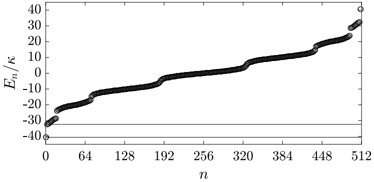
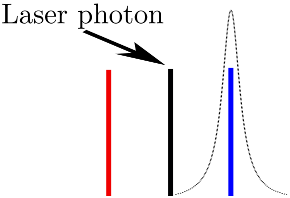
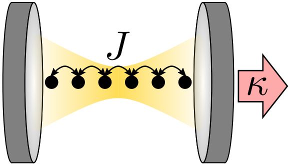




Δ_c = Gap, $\kappa \ll$ Gap

Example 2: Coupled spins to cavity



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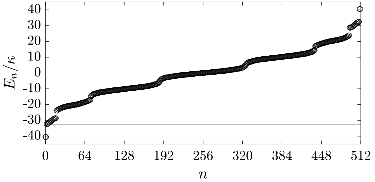
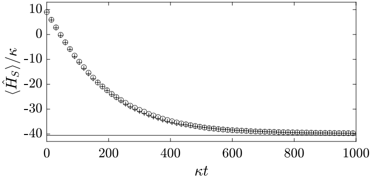
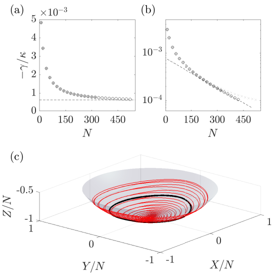


$\Delta_c = \text{Gap}$, $\kappa \ll \text{Gap}$



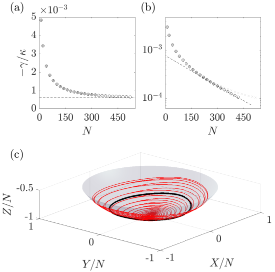
Cooling to ground state

Conclusion



- Description for time-periodic dissipative Dicke model
  - → Description of exponentially closing gap
  - → New insights into stabilization mechanism
    - S. B. Jäger et al., Phys. Rev. A 110, L010202 (2024).

Conclusion



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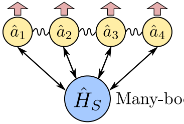

- Effective master equation for spins coupled by bosons
  - $\rightarrow$ Description of mediated interactions + dissipation
  - $\rightarrow$ More efficient description

S. B. Jäger et al., Phys. Rev. Lett. **129**, 063601 (2022).

What this method can(not) do

Can do:

- Description of trapping and cooling
- Description of interactions and correlated dissipation
  e.g. Superradiance, Subradiance, Dicke, Self-organization
- Dynamical phases, Limit cycles, and time crystals (?!)

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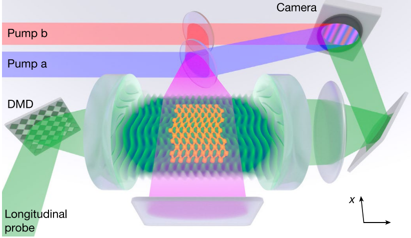
Cannot do:

- Everything that requires beyond Coupling^2 effects
  e.g. Higher atom-cavity systems with higher symmetries!



An optical lattice with sound

Y. Guo, R. M. Kroeze, B. P. Marsh, S. Gopalakrishnan, J. Keeling, B. L. Lev, Nature **599**, 211 (2021).

PHYSICAL REVIEW RESEARCH **3**, L032016 (2021)

Letter

Atom-only theories for U(1) symmetric cavity-QED models

Roberta Palacino and Jonathan Keeling
SUPA, School of Physics and Astronomy, University of St Andrews, St Andrews KY16 9SS, United Kingdom