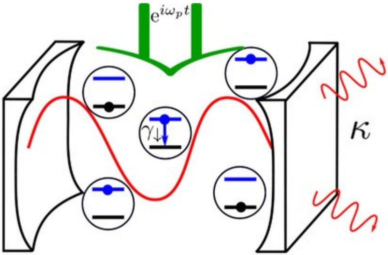
Bistability dynamics in the dissipative Dicke-Bose-Hubbard system



Tianyi Wu, **Sayak Ray** and Johann Kroha

Physikalisches Institut

RHEINISCHE FRIEDRICH-WILHELMS-UNIVERSITÄT BONN

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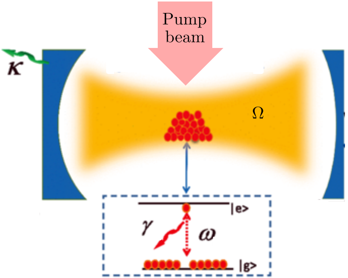
Outline

- Cold atoms in optical resonator
- Correlated phases in Dicke-Hubbard model
- Bistability and switching dynamics
- Summary and Outlook

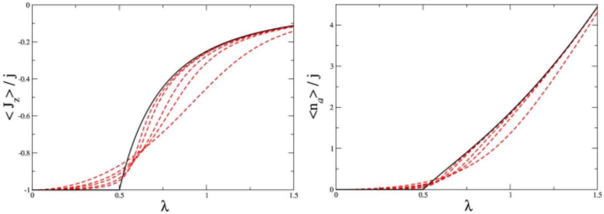
Two-level atoms in optical cavity

- Spins in single mode cavity:

$$
\begin{aligned}
\hat{H}_{\mathrm{D}} &= \Omega \hat{a}^\dagger \hat{a} + \frac{\mathcal{E}}{2} \sum_{r=1}^{N} \hat{\sigma}_z^r \\
&+ \frac{g}{\sqrt{N}} \sum_{r=1}^{N} \left(\hat{\sigma}_+^r \hat{a} + \hat{\sigma}_-^r \hat{a}^\dagger \right) \\
&+ \frac{\tilde{g}}{\sqrt{N}} \sum_{r=1}^{N} \left(\hat{\sigma}_-^r \hat{a} + \hat{\sigma}_+^r \hat{a}^\dagger \right)
\end{aligned}
$$



- Dicke QPT $(g = \tilde{g})$: normal $\rightarrow$ superradiant state at $g_c = \sqrt{\Omega\omega}/2$.



R. H. Dicke, *Phys. Rev.* **93**, 99 (1954); C. Emary and T. Brandes, PRL **90**, 044101 (2003).

Realization of the Dicke model

- Momentum states of each atom are mapped to atomic excitation.

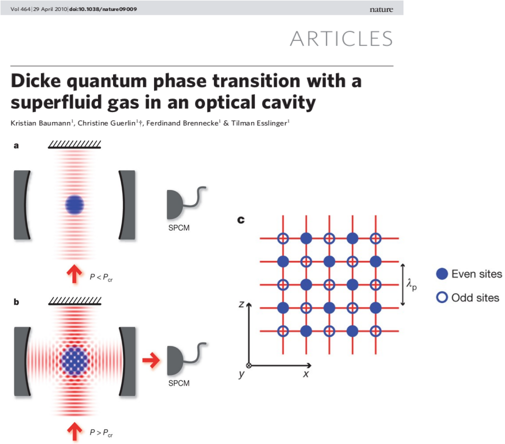
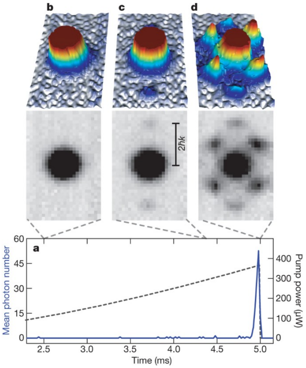


Figure 3: Observation of the phase transition.



- Dicke superradiance is signalled by onset of self-organized BEC.

K. Baumann, C. Guerlin, F. Brennecke and T. Esslinger, Nature (London) ***464****, 1301 (2010)*

Non-equilibrium phenomena with Dicke model

PNAS

Dynamical phase transition in the open Dicke model

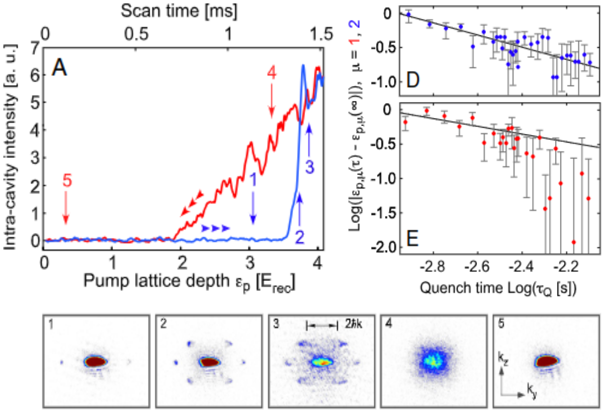
Jens Klinder, Hans Keßler, Matthias Wolke, Ludwig Mathey, and Andreas Hemmerich[1]

Institut für Laser-Physik, Universität Hamburg, 22761 Hamburg, Germany

Edited by Peter Zoller, University of Innsbruck, Innsbruck, Austria, and approved January 15, 2015 (received for review September 4, 2014)

The Dicke model with a weak dissipation channel is realized by coupling a Bose–Einstein condensate to an optical cavity with ultra-narrow bandwidth. We explore the dynamical critical properties of the Hepp–Lieb–Dicke phase transition by performing quenches across the phase boundary. We observe hysteresis in the transition

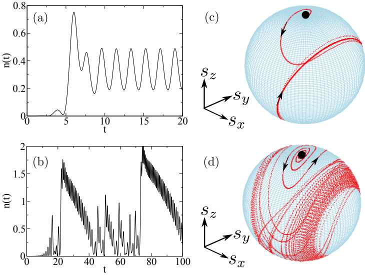
implemented experimentally by coupling a Bose–Einstein condensate (BEC) to a high-finesse resonator pumped by an external optical standing wave (32). A transition from a homogeneous phase (consisting of the condensate with no photons in the cavity) into a collective phase (with the atoms forming a density grating trapped



J. Klinder, H. Keßler, M. Wolke, L. Mathey, and A. Hemmerich, PNAS USA **112**, *3290 (2015)*

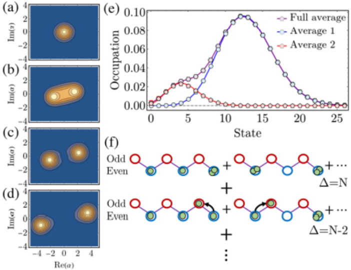
Non-equilibrium phenomena with Dicke model

Relaxation towards lasing



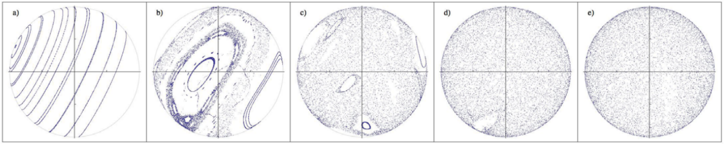
*<u>**S. Ray**</u>, A. Vardi, and D. Cohen, PRL **128**, 130604 (2022)*

Many-body self-organization



*C. Halati, A. Sheikhan, H. Ritsch, and C. Kollath, PRL **125**, 093604 (2020)*

Chaos and effective thermalization



*A. Altland and F. Haake, PRL. **108**, 073601 (2012)*

BECs in 2-D optical lattice coupled to cavity

LETTER

doi:10.1038/nature17409

Quantum phases from competing short- and long-range interactions in an optical lattice

Renate Landig[1], Lorenz Hruby[1], Nishant Dogra[1], Manuele Landini[1], Rafael Mottl[1], Tobias Donner[1] & Tilman Esslinger[1]

Insights into complex phenomena in quantum matter can be gained from simulation experiments with ultracold atoms, especially in cases where theoretical characterization is challenging. However, these experiments are mostly limited to short-range collisional

a stack of about 60 weakly coupled two-dimensional (2D) layers. These 2D layers are then exposed to a square lattice in the x–z plane formed by one free space lattice and one intracavity optical standing wave, both at a wavelength of $\lambda = 785.3$ nm. They create periodic optical

$\boxed{\text{New things in two dimensions}}$

- Physics of strong correlation in lattice.
- Off-diagonal long-range order exists at zero temperature.
- Extended Bose-Hubbard model including cavity-atom interactions with $\mathcal{Z}_2$ symmetry

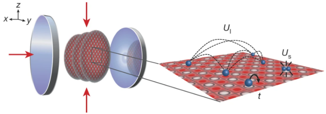
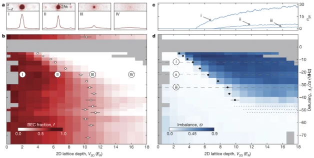


Figure 2: Characterization of the phases.



R. Landig et al, Nature (London) **532**, *476 (2016).*

Dicke-Bose-Hubbard model in two-dimensions

- Hamiltonian: $\hat{H} = \hat{H}_{\mathrm{a}} + \hat{H}_{\mathrm{c}} + \hat{H}_{\mathrm{ac}}$

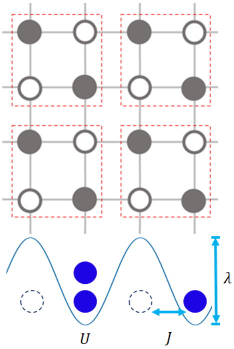
  Cavity: $\hat{H}_{\mathrm{c}} = \Omega \hat{a}^\dagger \hat{a}$

  BHM: $\hat{H}_{\mathrm{a}} = -J \sum_{\langle i,j \rangle} (\hat{b}_i^\dagger \hat{b}_j + \text{h.c.})$

  $$+ \sum_i \left[-\mu \hat{n}_i + \frac{U}{2} \hat{n}_i (\hat{n}_i - 1) \right]$$

  Coupling: $\hat{H}_{\mathrm{ac}} = -\frac{\lambda}{\sqrt{L}} (\hat{a} + \hat{a}^\dagger) \sum_i (-1)^i \hat{n}_i$

- Dissipative dynamics: $\boxed{\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] + \kappa \mathcal{L}[\hat{a}]}$



- Cluster mean-field Hamiltonian: $\hat{H} = \hat{H}_{\mathrm{c}} + \sum_l \hat{H}_{C_l}$

$$\hat{H}_{C_l} = \boxed{-J \sum_{\substack{\langle i,j \rangle \\ i,j \in C_l}} \hat{b}_i^\dagger \hat{b}_j - J \sum_{\substack{\langle i,j \rangle \\ i \in C_l, j \notin C_l}} \left(\hat{b}_i^\dagger \Phi_j + h.c. \right)}$$

$$+ \sum_{i \in C_l} \left[-\mu \hat{n}_i + \frac{U}{2} \hat{n}_i \left(\hat{n}_i - 1 \right) \right] - \frac{\lambda}{\sqrt{L}} (\hat{a} + \hat{a}^\dagger) \sum_{i \in C_l} (-1)^i \hat{n}_i$$

U. Pohl, ***S. Ray****, and J. Kroha, Ann. Phys. (Berlin) (2022)*

MF factorization and orderparameters

- $\hat{\rho} = \left(\prod_l \hat{\rho}_{C_l}\right) \times \hat{\rho}_{\mathrm{c}} \rightarrow \boxed{\dot{\hat{\rho}}_{\mathrm{c}} = -i[\hat{H}_{\mathrm{c}}^{\mathrm{MF}}, \hat{\rho}_{\mathrm{c}}] + \kappa \mathcal{L}[\hat{a}], \;\; \dot{\hat{\rho}}_{C_l} = -i[\hat{H}_{\mathrm{BHM}}^{\mathrm{MF}}, \hat{\rho}_{C_l}]}$

BHM: $$\hat{H}_{\mathrm{BHM}}^{\mathrm{MF}} = \boxed{-J \sum_{\substack{\langle i,j \rangle \\ i,j \in C_l}} \hat{b}_i^\dagger \hat{b}_j - J \sum_{\substack{\langle i,j \rangle \\ i \in C_l, j \notin C_l}} \left(\hat{b}_i^\dagger \Phi_j + h.c.\right)}$$
$$+ \sum\nolimits_{i \in C_l} \left[-\mu \hat{n}_i + \tfrac{U}{2} \hat{n}_i \left(\hat{n}_i - 1\right)\right] - \boxed{\lambda(\alpha + \alpha^*) \sum_{i \in C_l} (-1)^i \hat{n}_i}$$

Cavity: $$\hat{H}_{\mathrm{c}}^{\mathrm{MF}} = \Omega \hat{a}^\dagger \hat{a} - \boxed{\frac{\lambda}{\sqrt{L}} (\hat{a} + \hat{a}^\dagger) \sum_{i \in C_l} (-1)^i n_i}$$

- Orderparameters and characterization of various phases.

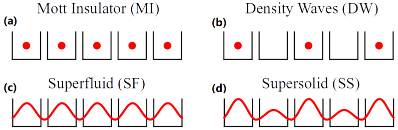
Dicke phases

Photon amplitude: $\alpha = \langle \hat{a} \rangle / \sqrt{L}$
Photon number: $n_{\mathrm{P}} = \langle \hat{a}^\dagger \hat{a} \rangle / L$

condensate phases

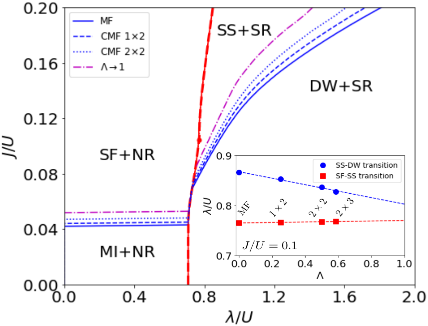
condensate amp.: $\Phi_{\mathrm{e,o}} = \langle \hat{b}_{\mathrm{e,o}} \rangle$
boson density: $n_{\mathrm{e,o}} = \langle \hat{n}_{\mathrm{e,o}} \rangle$

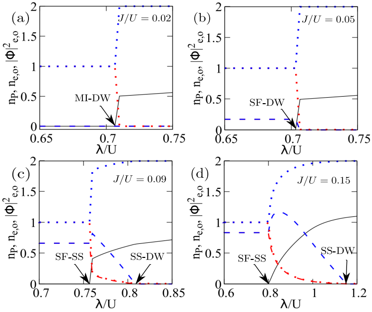
Schematics of various atomic phases



Ground state phase diagram at zero temperature

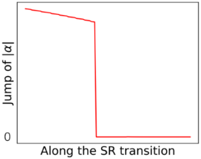
- Phase diagram for $\langle \hat{n} \rangle = 1$ with $\kappa = 0$.





- $\Lambda = \frac{N_b}{N_c z_c/2}$, $N_{c,b}$: size & bonds in cluster, z_c: co-ordination no.
- Dicke transition is discontinuous below $(J_c/U, \lambda_c/U) \approx (0.105, 0.77)$.
- Consistent with QMC and B-DMFT in equilibrium.

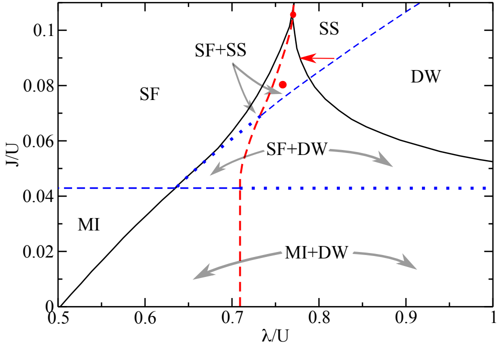
Dicke transition with jump



Masterarbeit in Physik, T. Wu (2023)

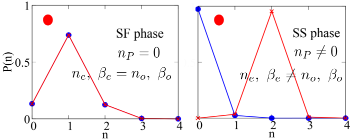
Bistability and co-existence of phase

Coexistence of phases for density $\langle \hat{n}_e \rangle + \langle \hat{n}_o \rangle = 2$ and for 1×2 cluster

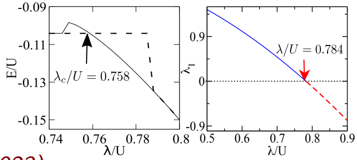


- Dashed: QPT (gr. state).
- Dotted: QPT lines extended to bistability.
- Solid: bistability border.
- Red(blue): jump(cont.).

SF-SS coexistence



Energy diff & SF-instability

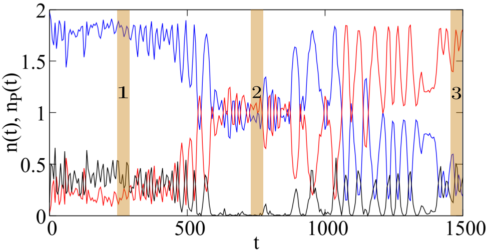


- Bistabilities between condensate phases and insulating phases are observed.
- The bistability boundaries meet at critical J_c/U, where Dicke transition is continuous.

T. Wu, S. Ray, and J. Kroha, arXiv:2311.13301 (2023)

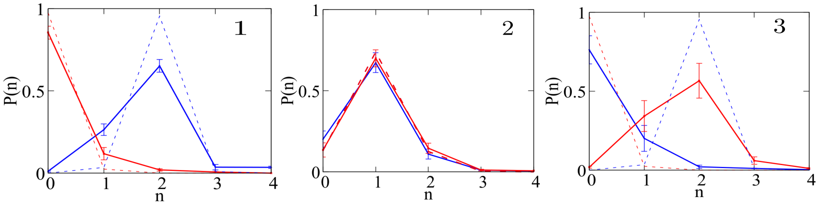
Switching dynamics in bistability

Oscillations between SF and SS phases



- At $t = 0$ the system is in SS ground state.
- Excess energy is provided by photon field: $\alpha(t=0) = \alpha_{\mathrm{GS}} + \delta\alpha$.
- Unitary time-evolution is performed for $\kappa = 0$.

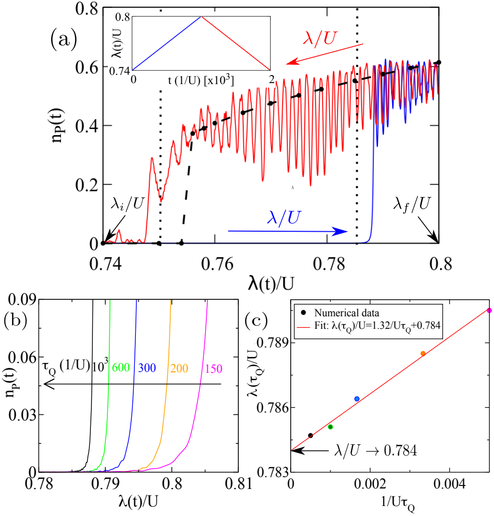
Local number state distribution of atoms



- $P_{o(e)}(n) = \langle n | \mathrm{Tr}_{e(o)} \hat{\rho}_{C_l} | n \rangle$, $|n\rangle$ is the number state of bosons.

Observation of hysteresis

Linear ramp in λ across the discontinuous transition from SF to SS



- At $t = 0$ the system is in SF ground state, $\alpha = 0$.
- Linear ramp in $\lambda(t)$: $0.74 \leftrightarrow 0.8$ ($U\tau_Q = 10^3$).
- Dashed line: n_P for GS (jump at QPT).
- Dotted line: Bistability border obtained from numerical self-consistency.

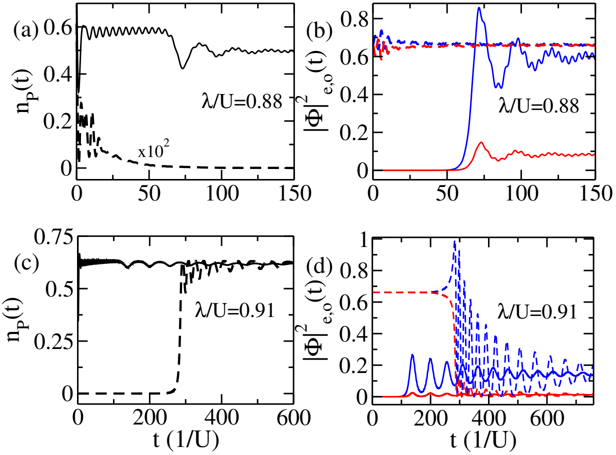
τ_Q-dependence

- Increasing $\tau_Q \to$ dynamics is more adiabatic
- $\tau_Q \to \infty$: transition agrees with bistability boundary.

T. Wu, S. Ray, and J. Kroha, arXiv:2311.13301 (2023)

Relaxation dynamics with finite dissipation

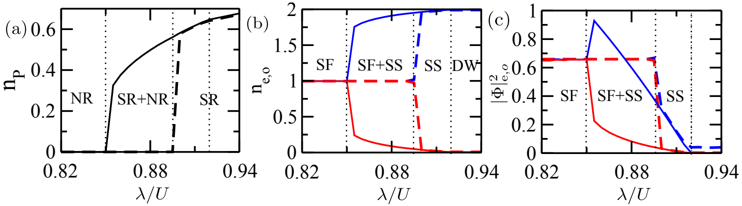
- Time-evolution of orderparameters with different initial preparations. Parameters: $J/U = 0.09$ and $\kappa/U = 1.08$.



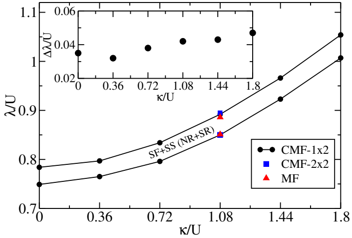
- Dashed and solid lines → initial SF and DW states, respectively.

Steady states and bistability with finite dissipation

- Orderparameters vs λ in steady states for $J/U = 0.09$ and $\kappa/\Omega = 1.08$



Bistability boundary at finite κ/U

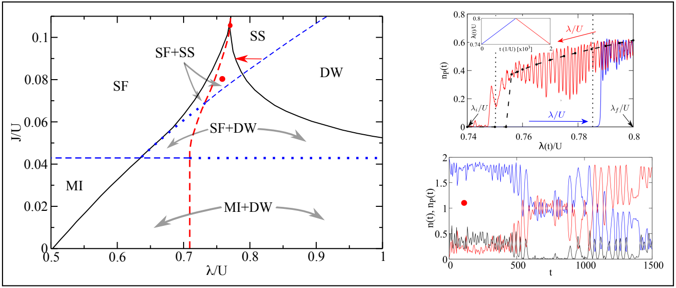


- Coexistence region for several values of κ/U.
- Bistability occurs at a higher λ/U with finite κ/U.
- Width of coexistence region $\Delta\lambda/U$ is less sensitive to κ/U.

T. Wu, S. Ray, and J. Kroha, arXiv:2311.13301 (2023)

Summary

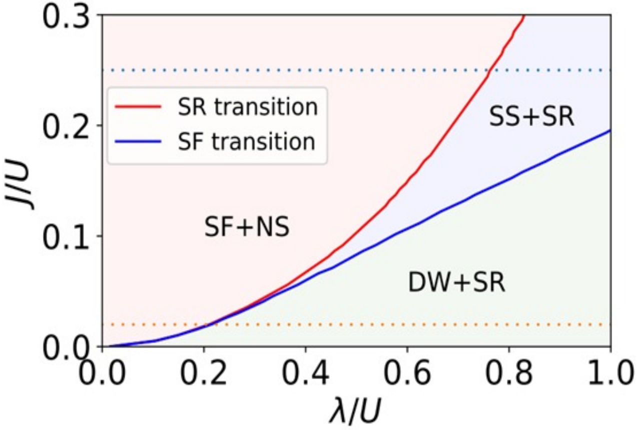
- Bistability and dissipative dynamics of BEC in Dicke-Bose-Hubbard model
  - Ground state phase diagram at $T = 0$ and at $\kappa = 0$
  - CMF for time evolution in long-range interacting system
  - Discontinuous behaviour at Dicke transition and several coexistence regions (survives under dissipation)



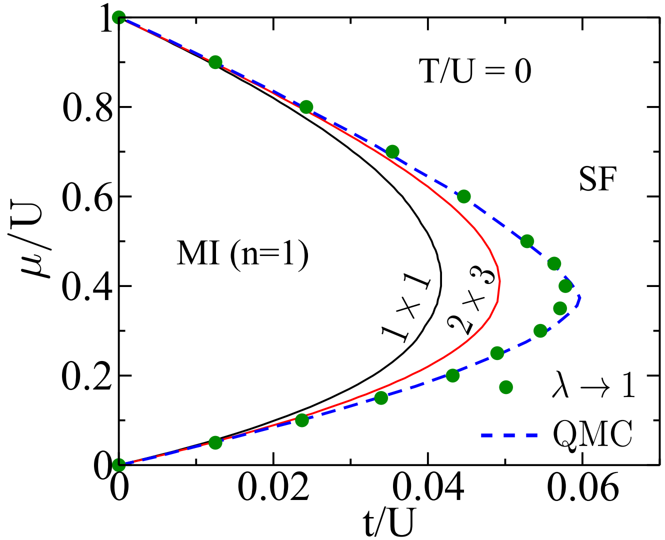
  - Switching dynamics and hysteresis behavior in bistability

THANK YOU

Phase diagram with $\langle \hat{n} \rangle = 1/2$

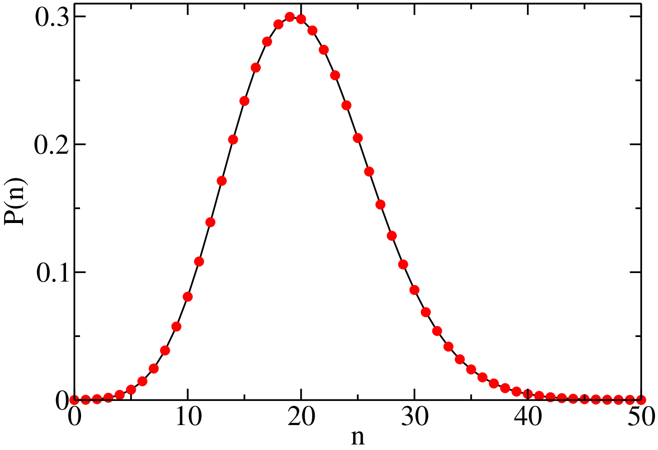


BHM phase diagram (CMF vs QMC)



Number state distribution of photon wavefunction

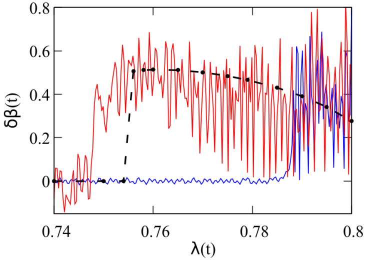
- The parameters are: $J/U = 0.08$ and $\lambda/U = 0.76$ for which the GS is SS phase.

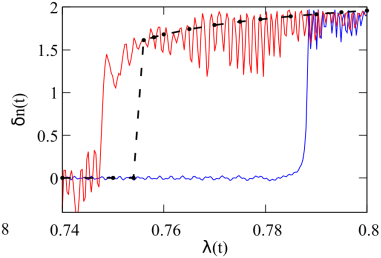


- Red dots: Coherent state of photon $|\psi\rangle_c = e^{-|\alpha|^2/2} e^{\alpha a^\dagger} |0\rangle$.
- Solid line: Photon wavefunction $|\psi\rangle$ in SS phase, and $\alpha = \langle\psi|\hat{a}|\psi\rangle$.

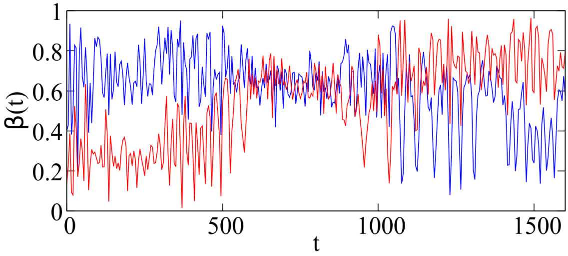
Dynamics of condensate in bistability

Dynamics of condensate and total boson density across SF-SS transition

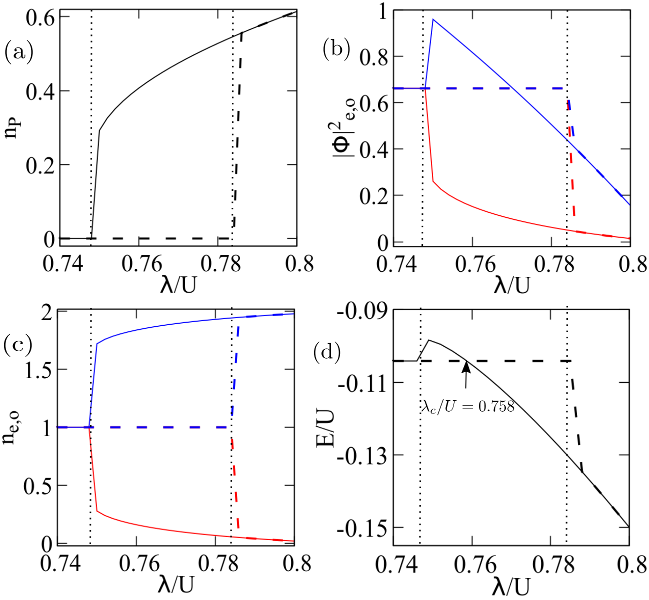




Oscillatory dynamics of condensate between SF and SS phases

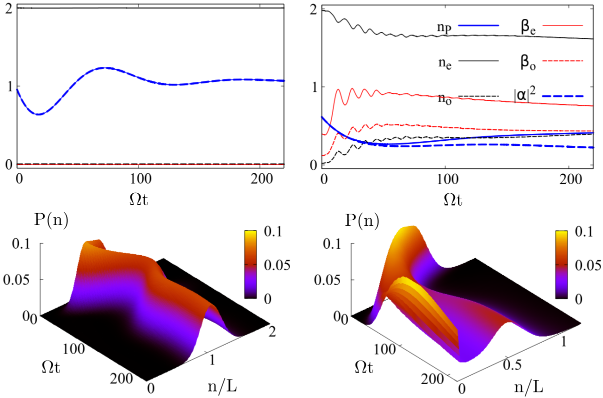


Bistability and orderparameters



Effect of atom-photon entanglement

- Initial state preparation: $|\psi(t=0)\rangle = |\psi\rangle^{1\times 2}_{\mathrm{DW/SS}} \bigotimes |\psi\rangle_c$. Coherent state of photon: $|\psi\rangle_c = e^{-|\alpha|^2/2} e^{\alpha a^\dagger} |0\rangle$



- Longer relaxation time, tunnelling between attractors.