Partial disorder
in two-dimensional spin systems

October, 2023 - Journal Club
~17 slides

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UNIVERSITÄT BONN

Partial Disorder (PD)

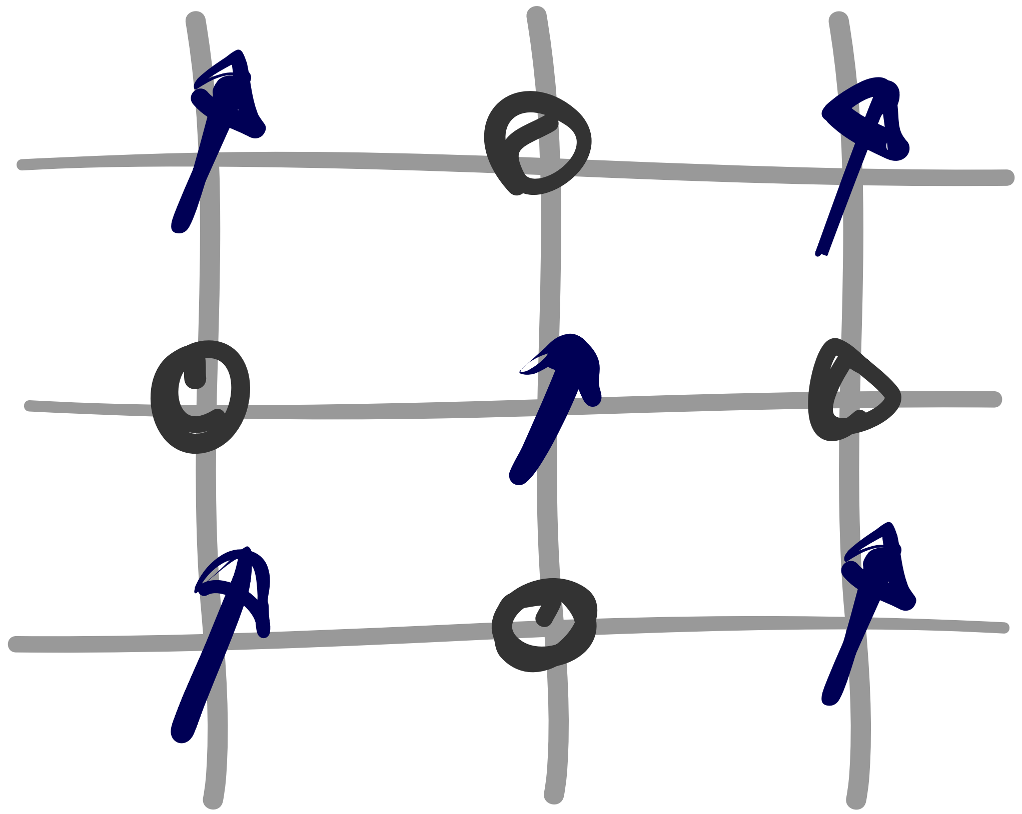
Coexistence of order and disorder

Ordered subsystem

- Algebraically decaying correlations
- Finite magnetization

Disordered subsystem

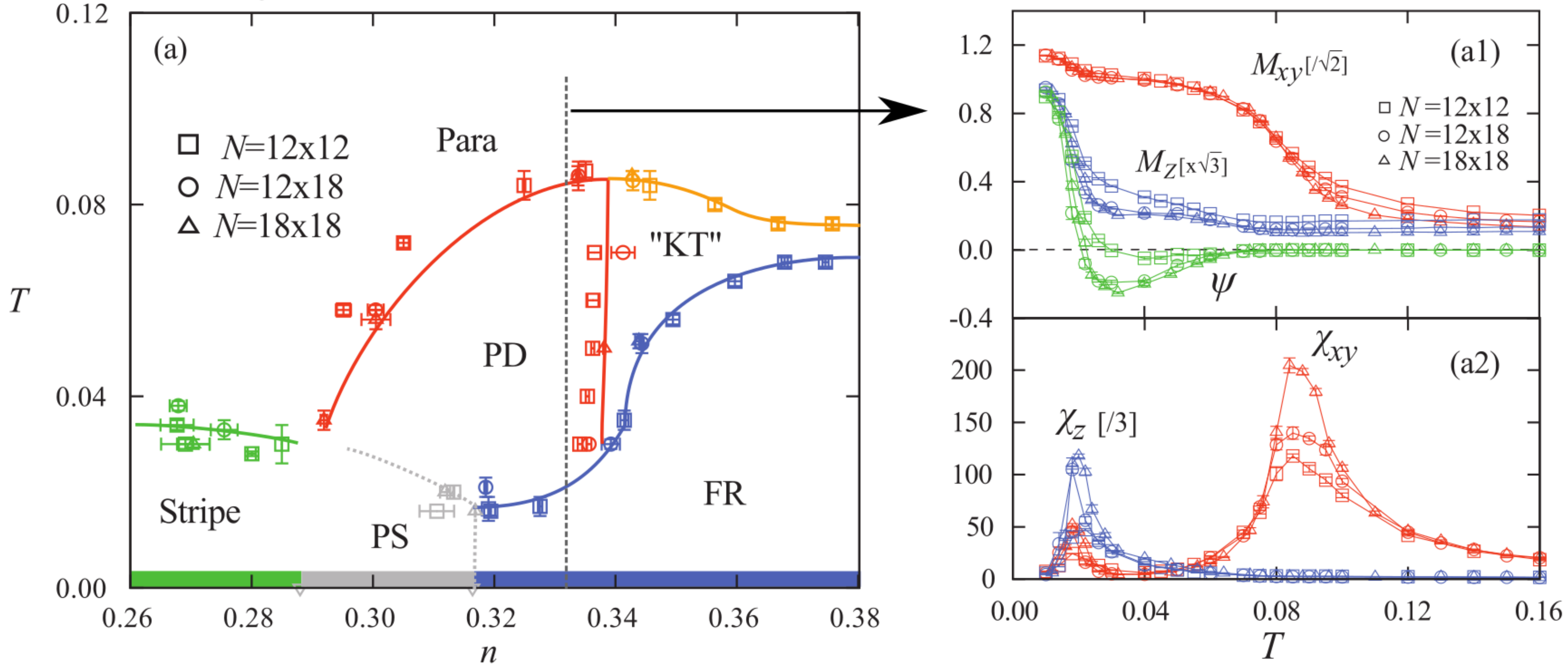
- Exponentially decaying correlations
- Ideal paramagnet?



PD at finite temperature

Kondo lattice model (triangular)

$$H = -t \sum_{\langle i,j \rangle,\sigma} (c^{\dagger}_{i\sigma} c_{j\sigma} + \mathrm{H.c.}) + J \sum_i \sigma_i^z S_i$$



Ishizuka and Motome, PRB 87, 155156 (2013)

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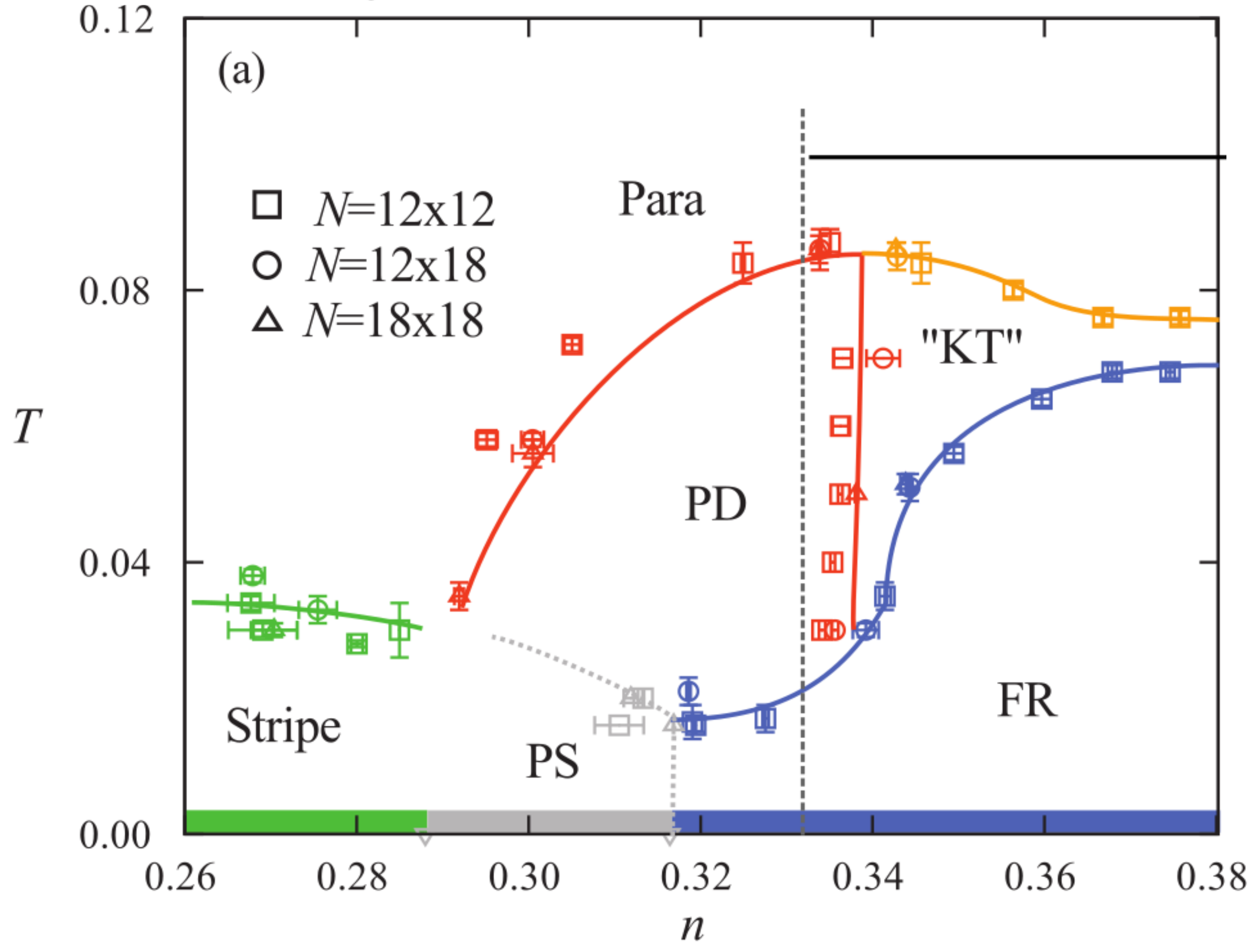


- AF + Ideal Paramagnet
- Finite temperature
- Strong anisotropy

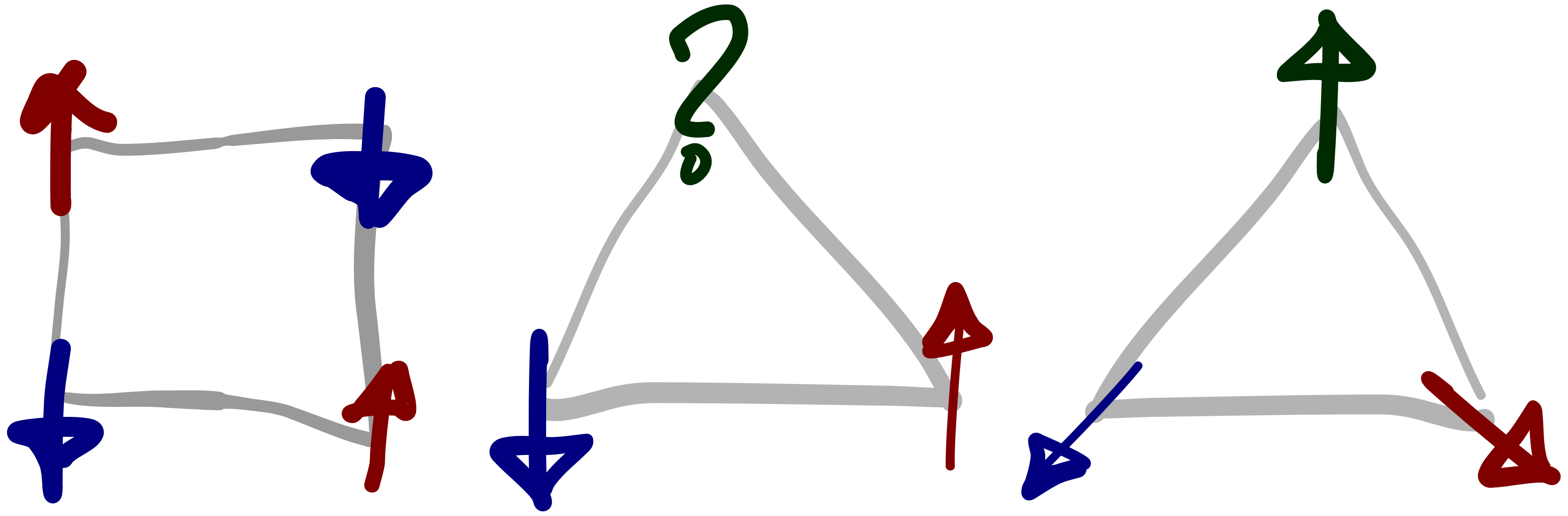
Ishizuka and Motome, PRB 87, 155156 (2013)

Heisenberg model and Frustration

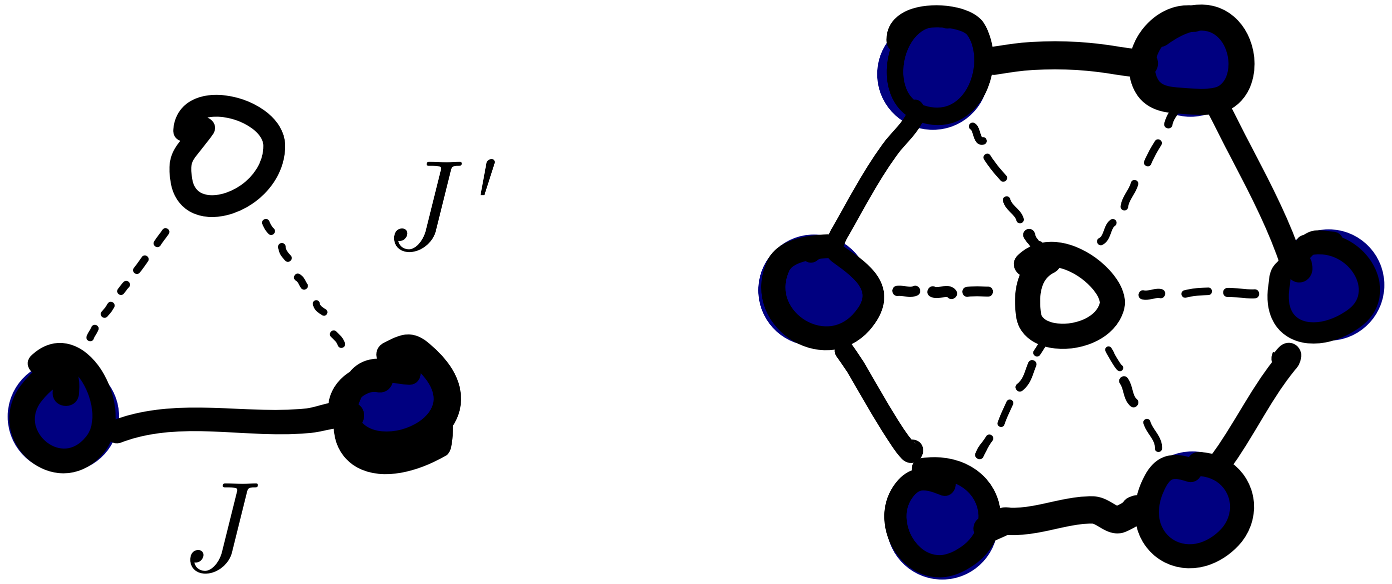
Heisenberg model

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{[ik]} \mathbf{S}_i \cdot \mathbf{S}_k$$

Magnetic Frustration



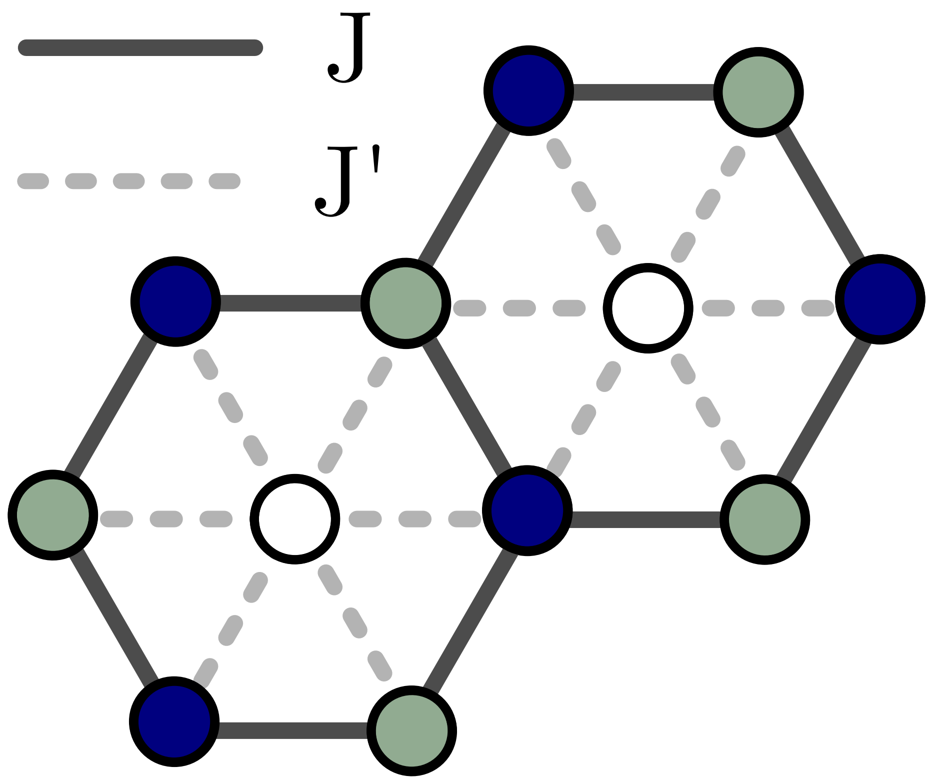
Toy models



$J_C{}'=J$ $\quad$ $J_C{}'=0.63J$

PD in the honeycomb lattice (at T=0)

Stuffed honeycomb lattice



Gonzalez et al., PRL 122, 017201 (2019)

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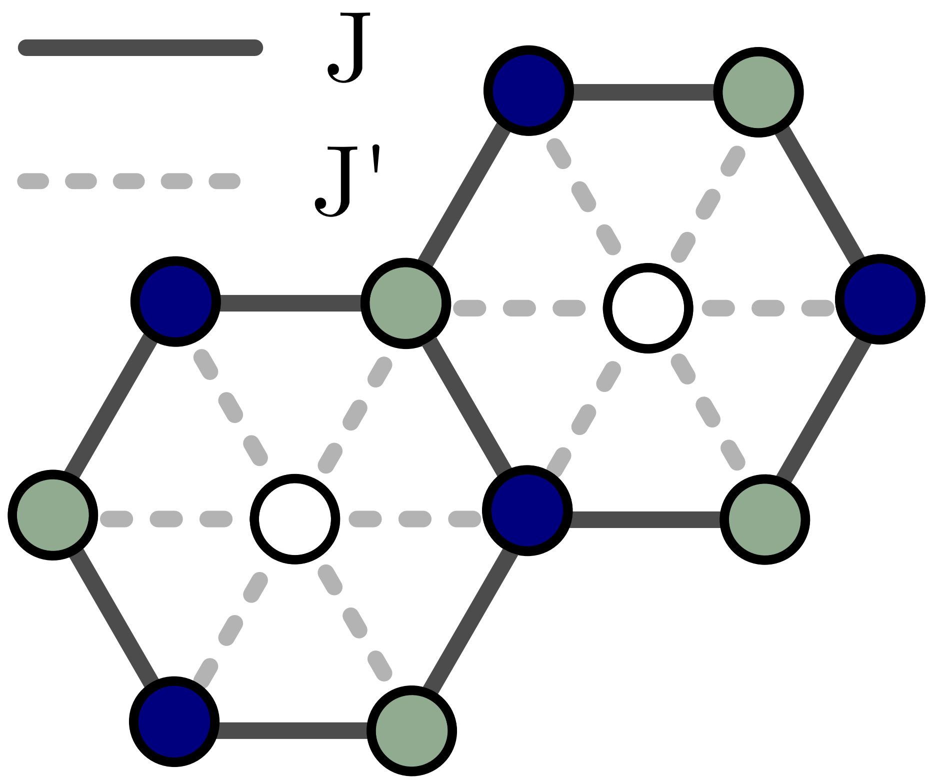
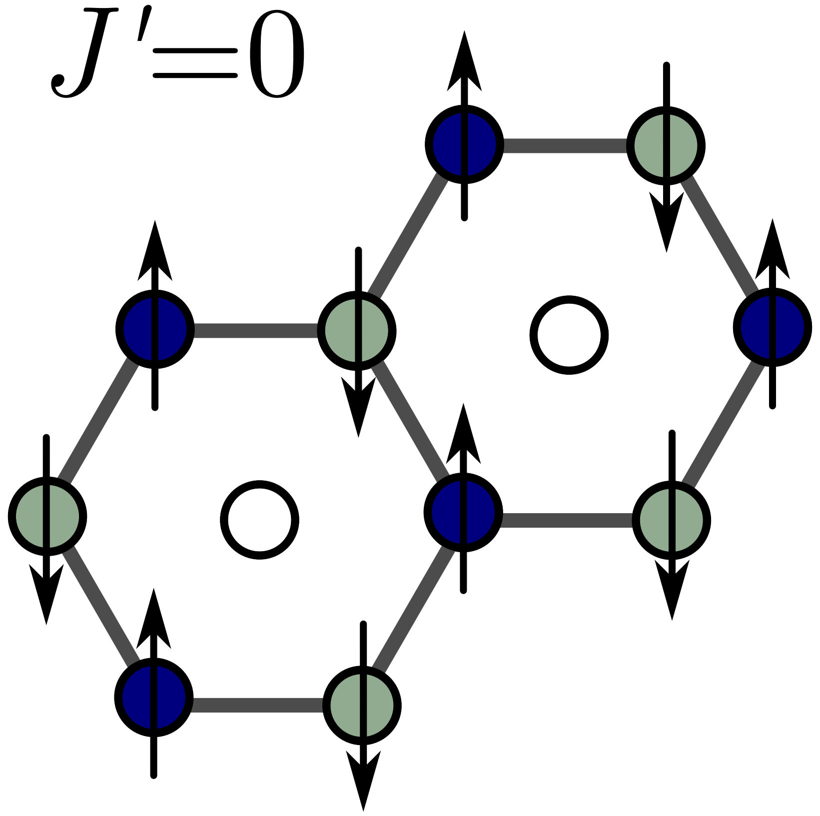
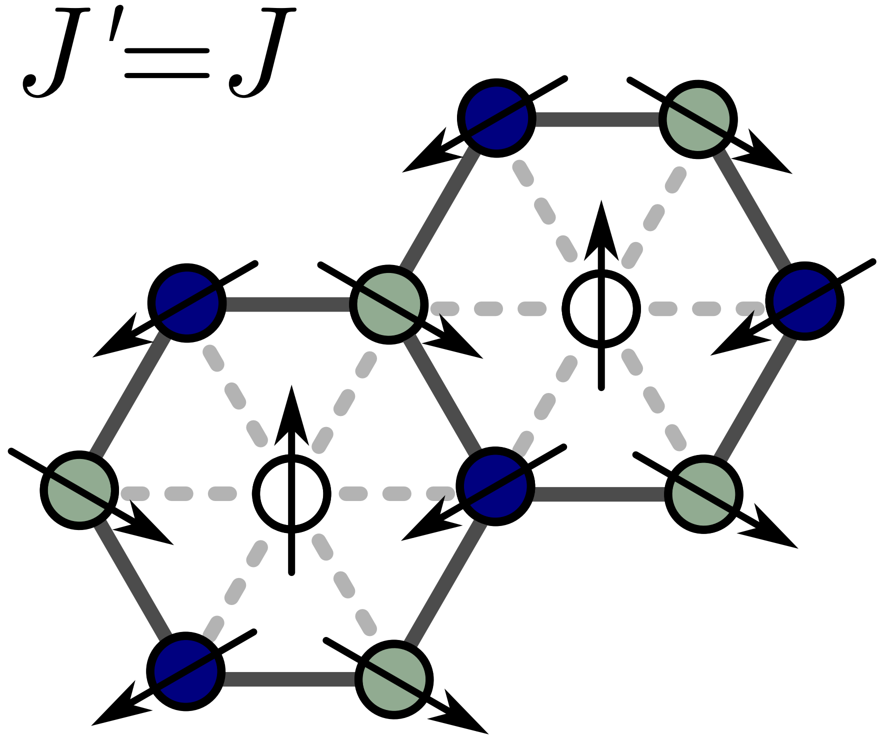


Known limits



180º Néel order

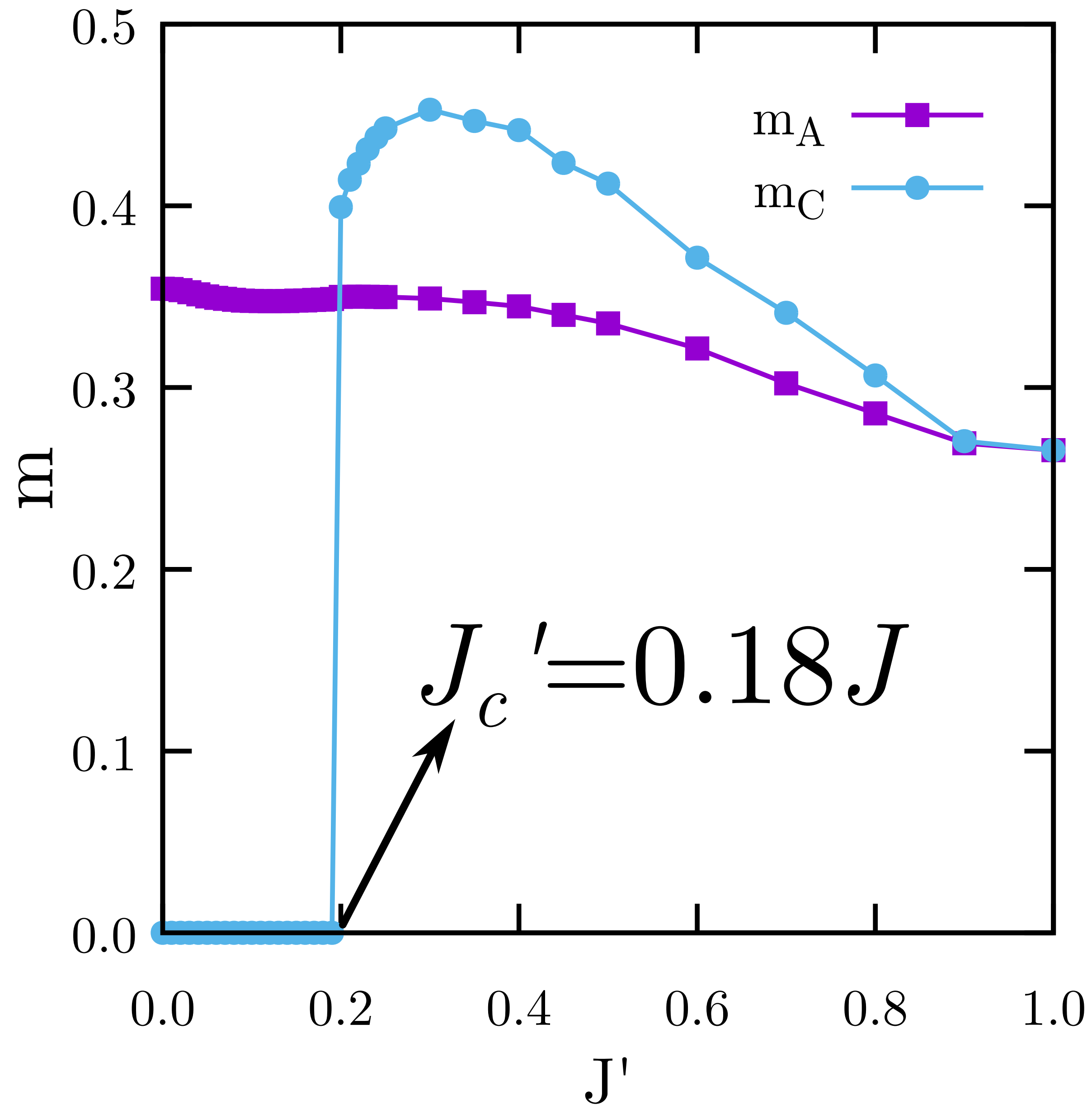


120º Néel order

Gonzalez et al., PRL 122, 017201 (2019)

PD in the honeycomb lattice (at T=0)

Magnetization



Gonzalez et al., PRL 122, 017201 (2019)

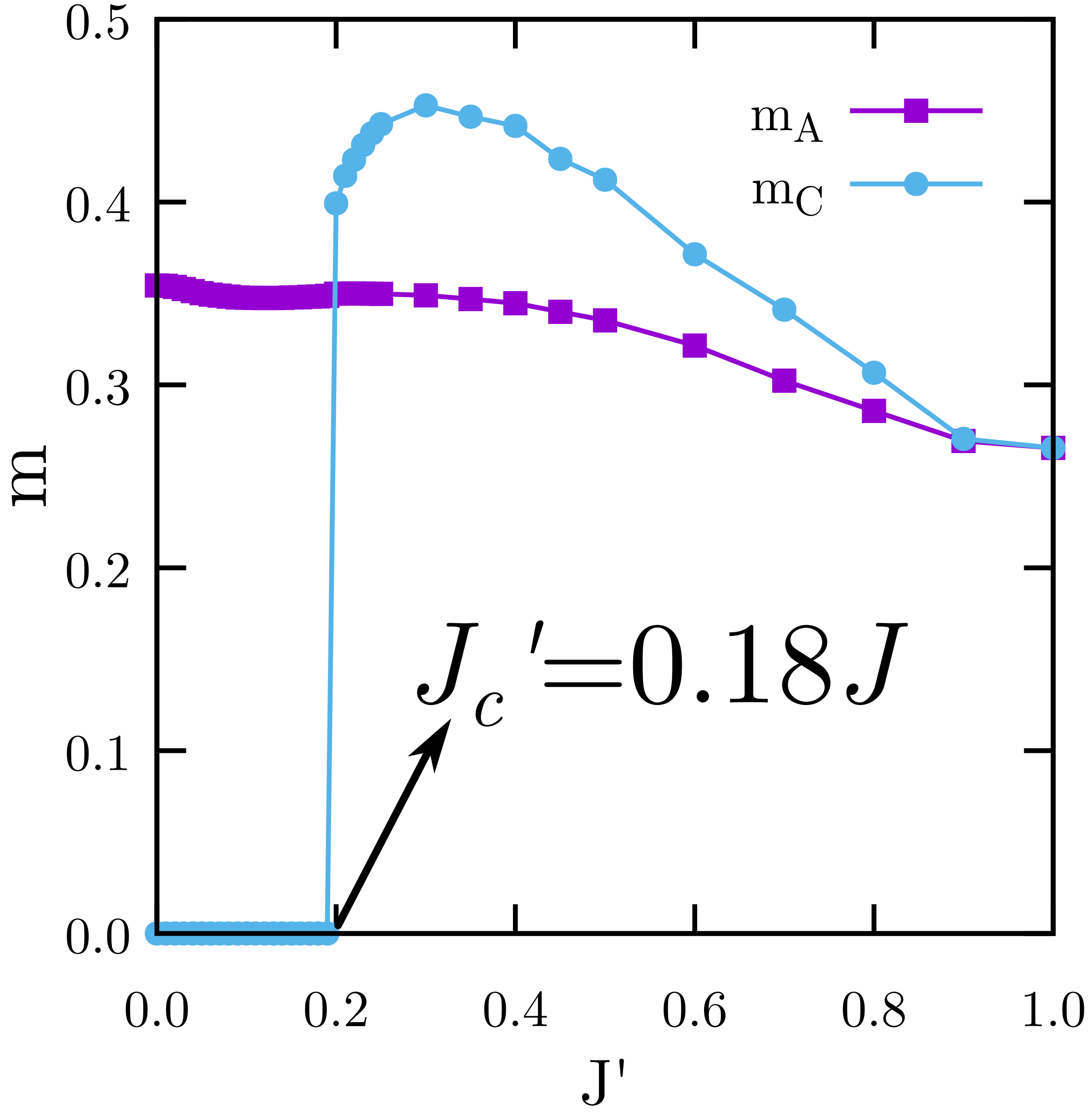
MPS simulations

- $L_y = 6$
- CBC, $L_x > L_y/2$
- D = 3000
- t.e. < 10^{-6}

$$m_\alpha^2 = \frac{1}{N_\alpha(N_\alpha - 1)} \sum_{\substack{i,j\in\alpha \\ i\neq j}} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle,$$

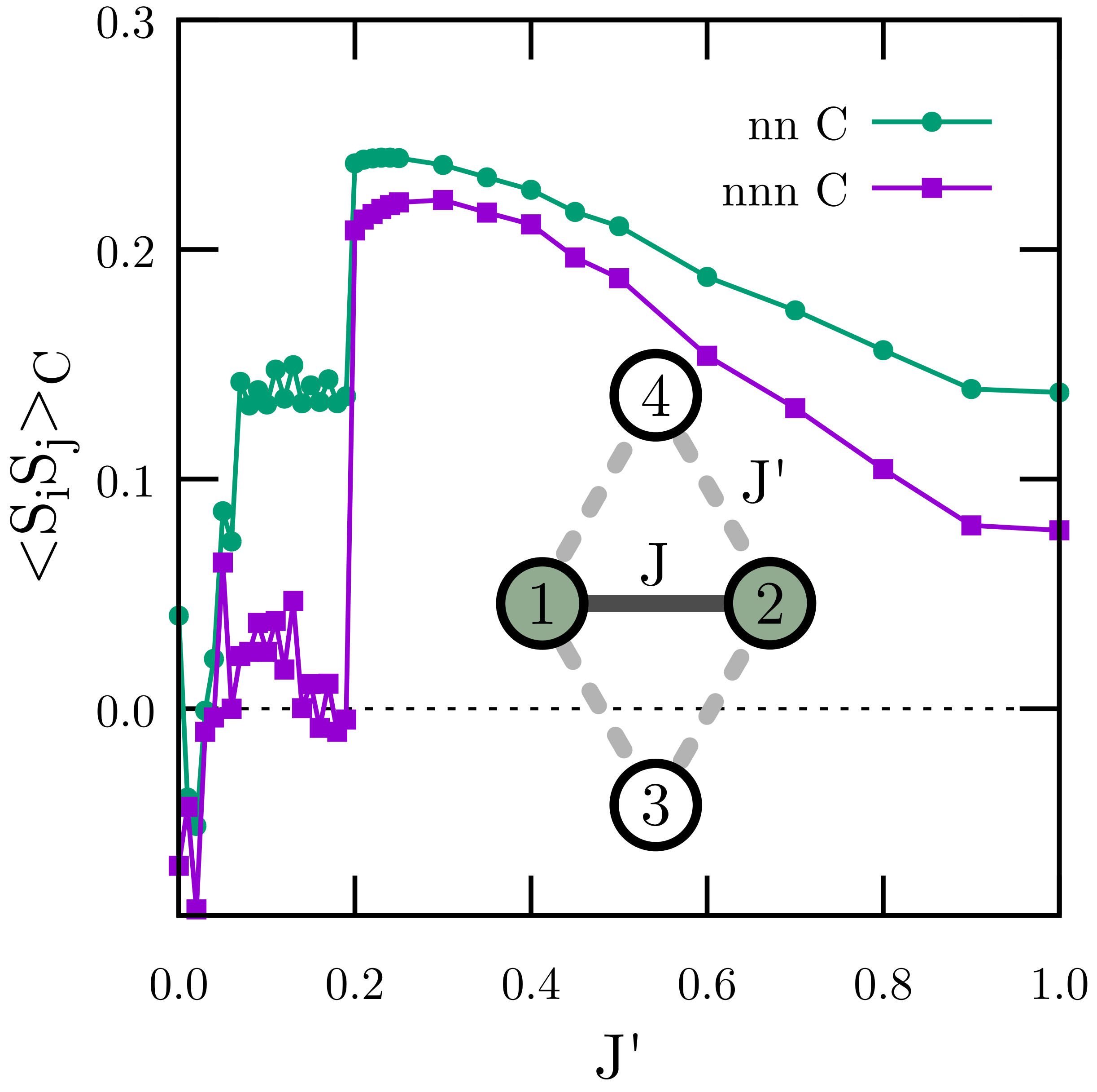
PD in the honeycomb lattice (at T=0)

Magnetization



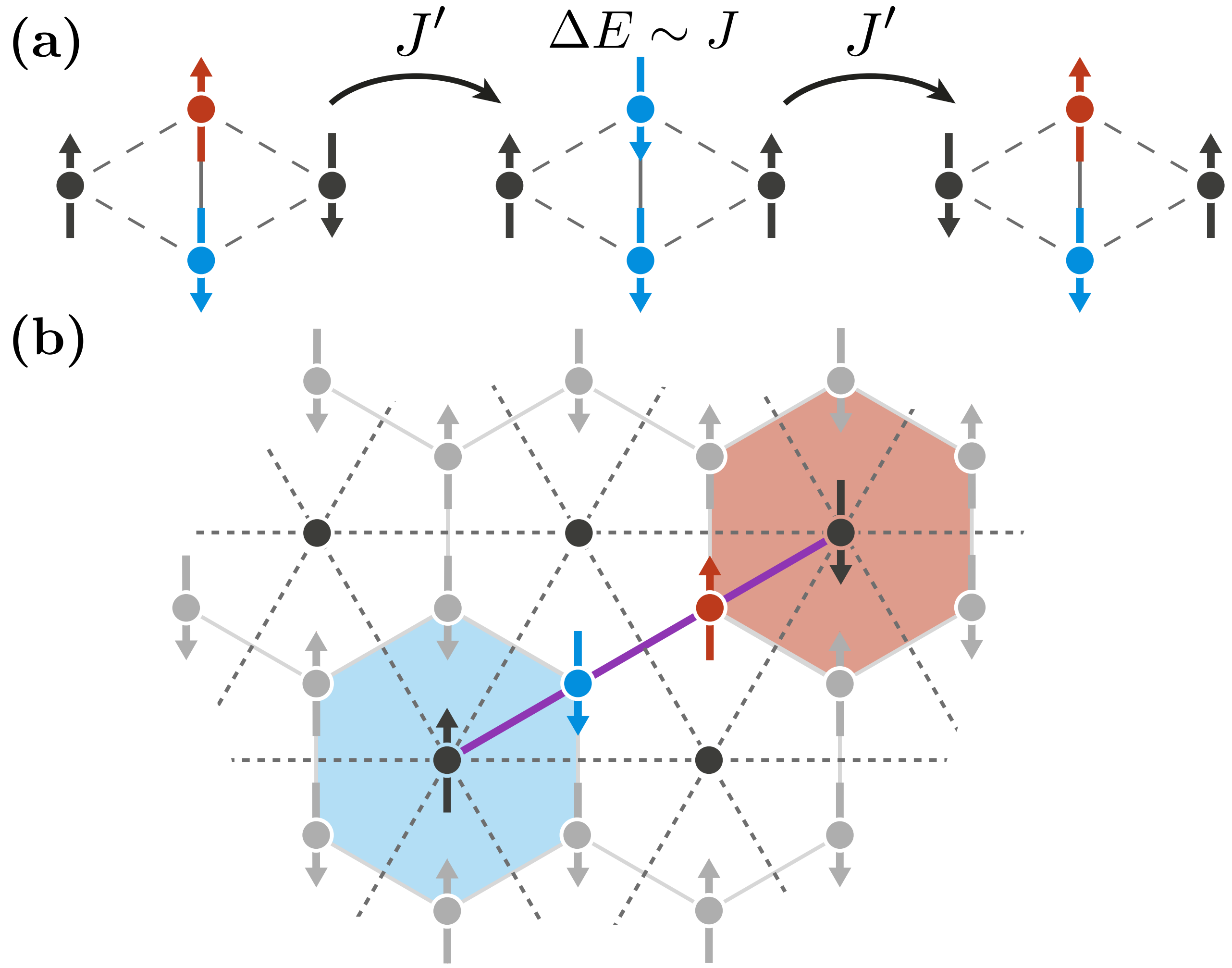
Gonzalez et al., PRL 122, 017201 (2019)

Finite correlations

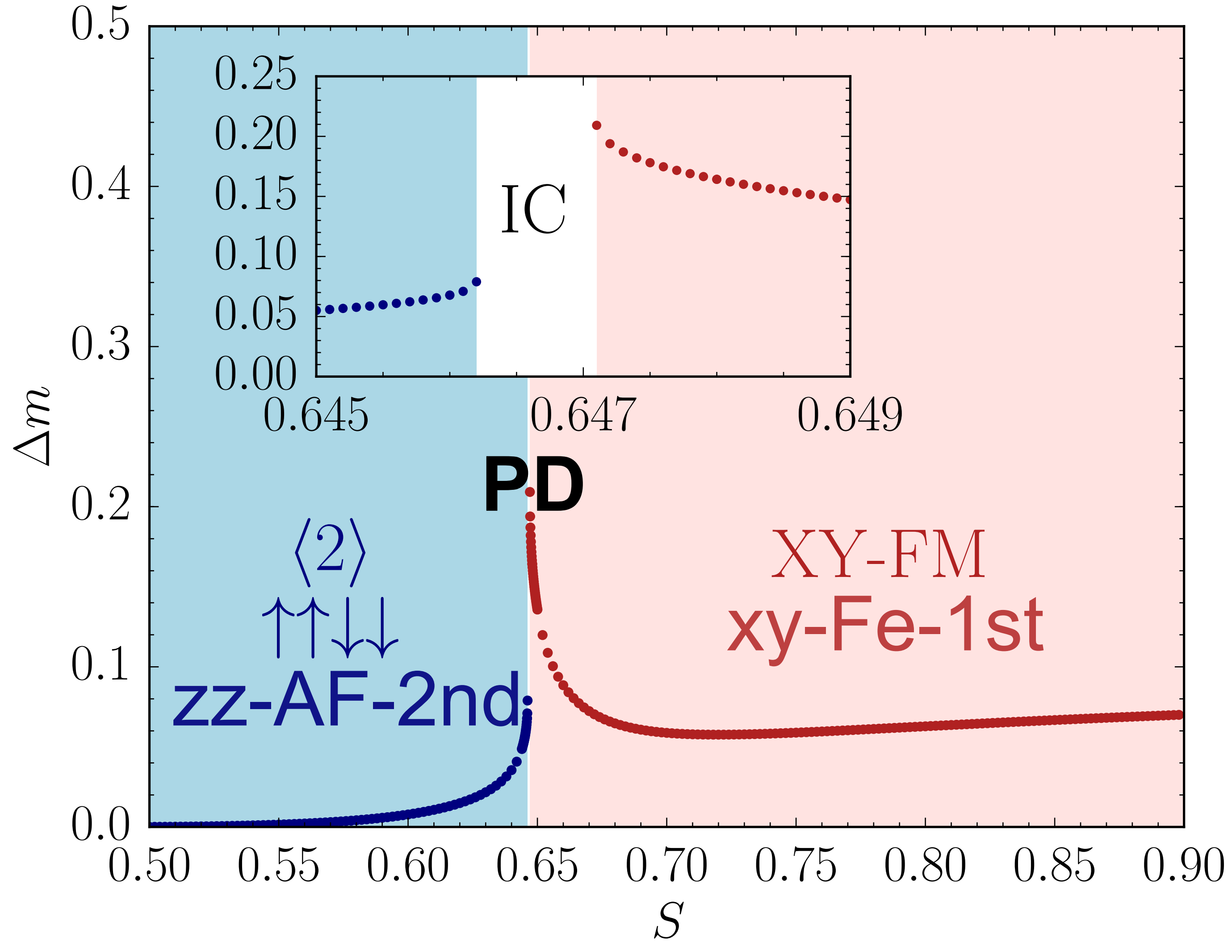


Effective model

Effective interactions



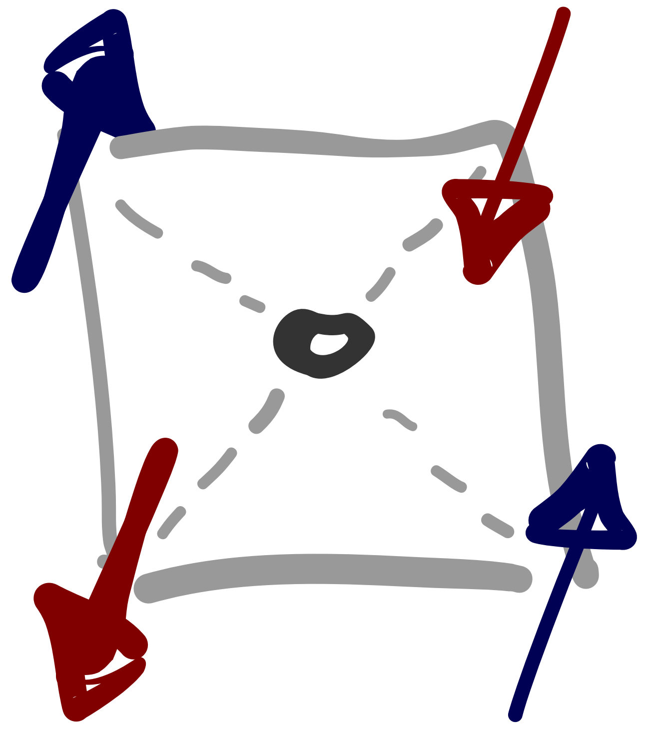
S-dependence



Seifert and Vojta, PRB 99, 155156 (2019)

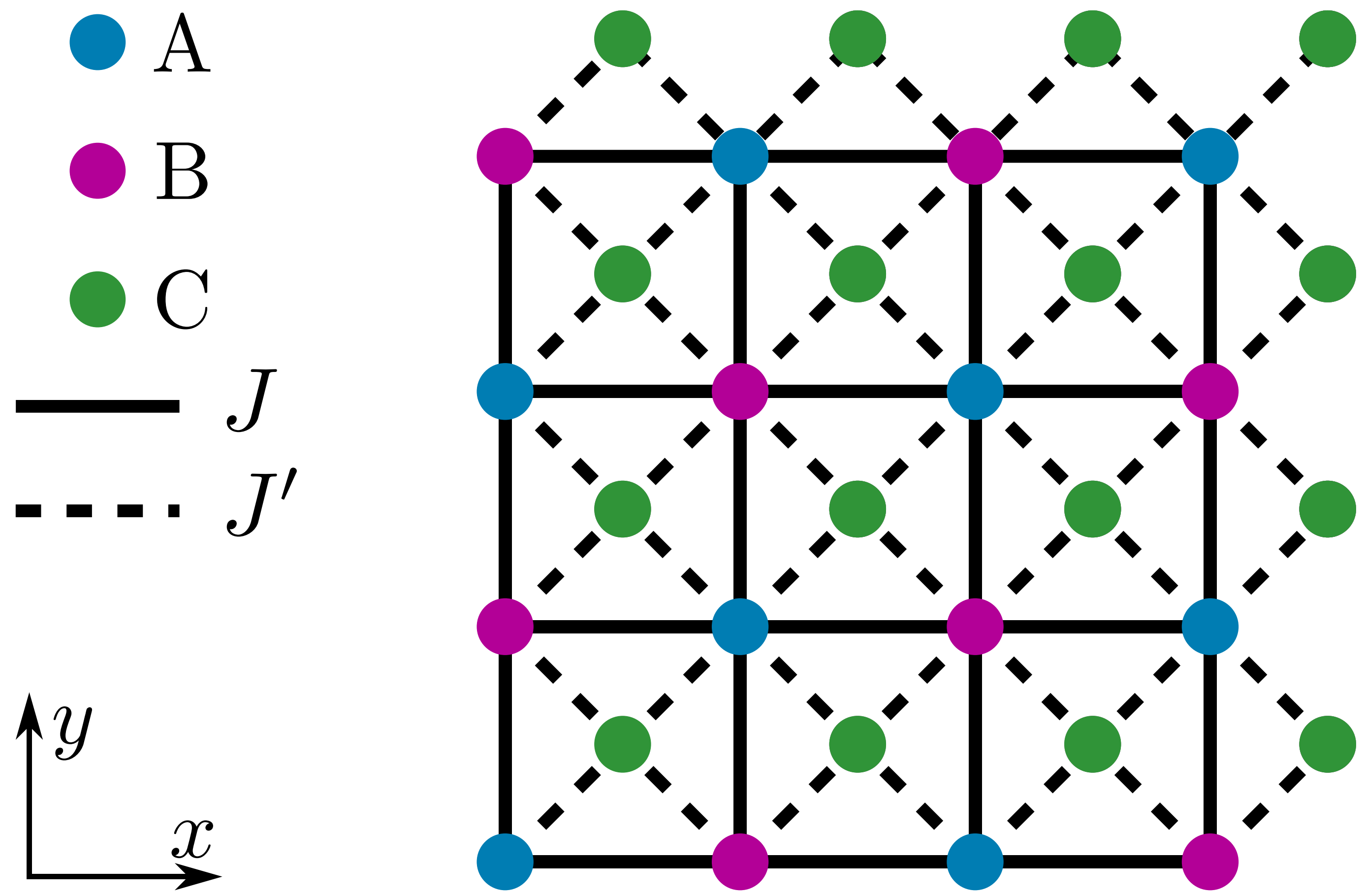
Stuffed square lattice
Motivation

- Is PD something more general?
- Which other systems can host PD?
- Can we "tune" the disordered state?



Another partially disordered phase?

Stuffed square lattice

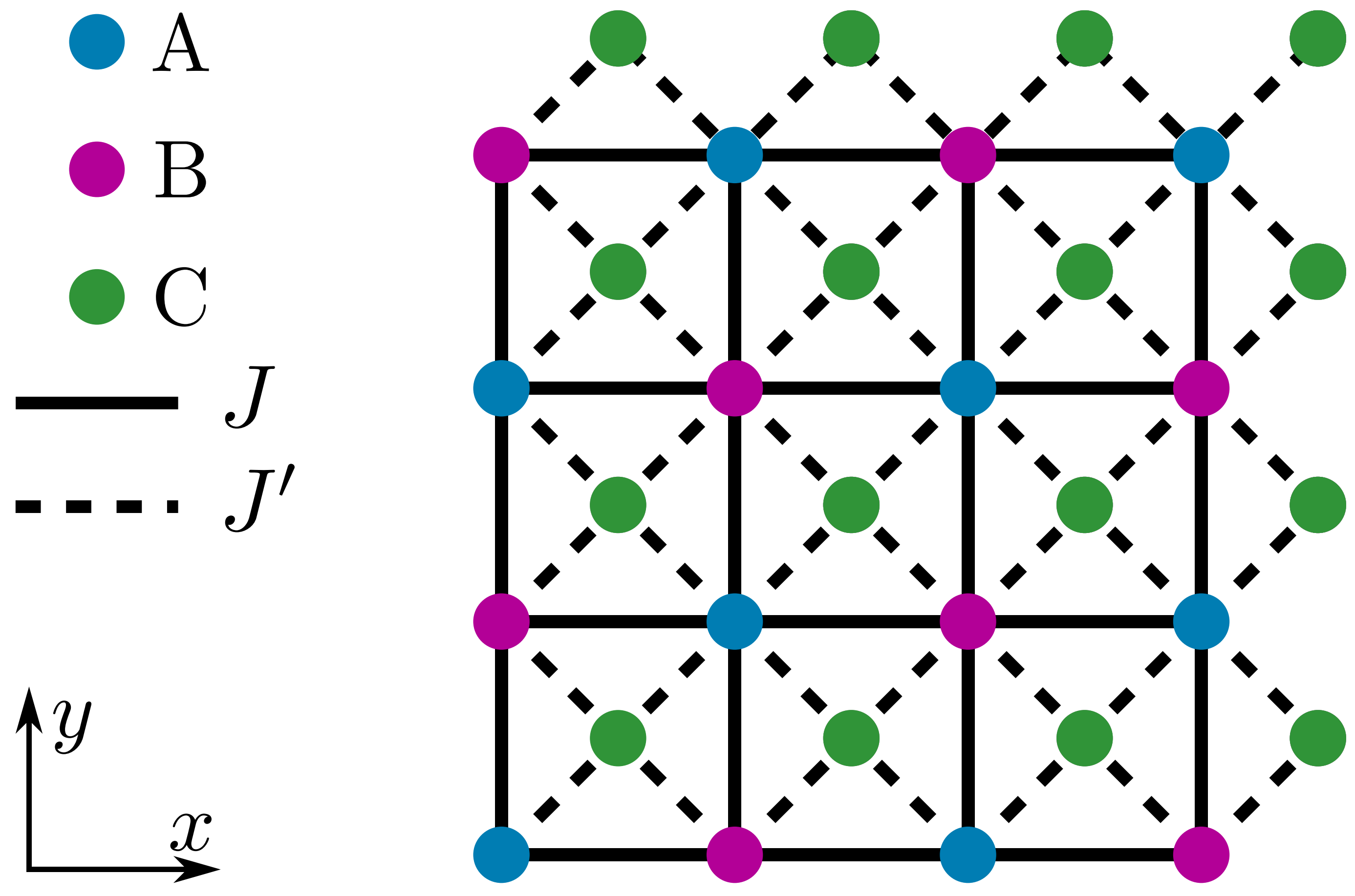


$$\mathcal{H} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{[ij]} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$J = \cos\left(\alpha \tfrac{\pi}{2}\right) \qquad J' = \sin\left(\alpha \tfrac{\pi}{2}\right)$$

Another partially disordered phase?

Stuffed square lattice



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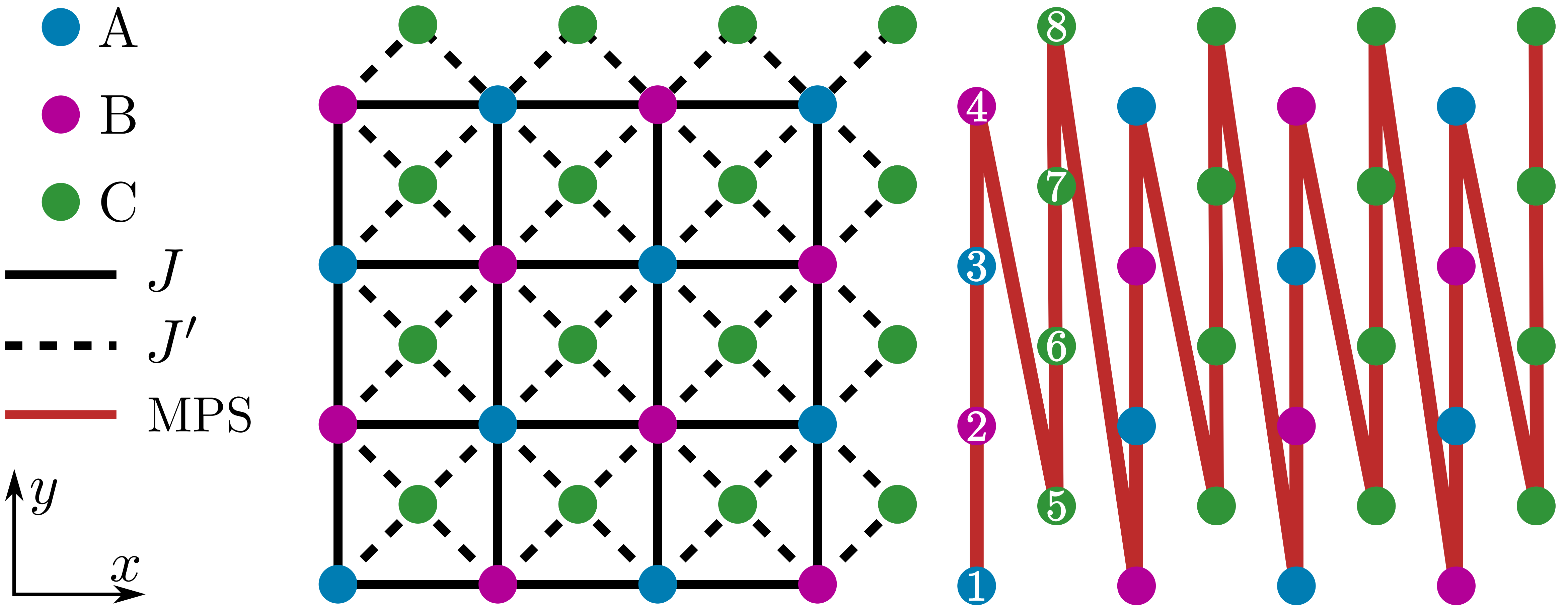
$$J = \cos\left(\alpha \tfrac{\pi}{2}\right) \qquad J' = \sin\left(\alpha \tfrac{\pi}{2}\right)$$

Static Structure Factor

$$S^X(\mathbf{q}) = \frac{1}{N_X} \sum_{i,j \in X} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle \, e^{i\mathbf{q}\mathbf{r}_{ij}}$$

$$m^X_{\mathrm{FE}} = \sqrt{\frac{S_X(\mathbf{0})}{N_X}} \qquad m^X_{\mathrm{AF}} = \sqrt{\frac{S_X(\boldsymbol{\pi})}{N_X}}$$

Numerical implementation



Itensor libraries

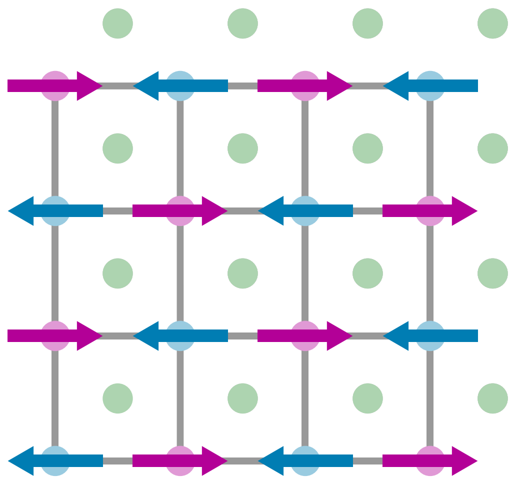
- L_y up to 12 (16)
- CBC, $L_x > L_y/2$
- D = 3000 (5000)
- t.e. < 10^{-6}
- GS calculations
- Correlations

Itensor: Fishman et al., SciPost Phys. Codebases, 4 (2022)

Known phases

$J = \cos\left(\alpha\frac{\pi}{2}\right) \qquad J' = \sin\left(\alpha\frac{\pi}{2}\right)$

α = 0.0 (J'=0)

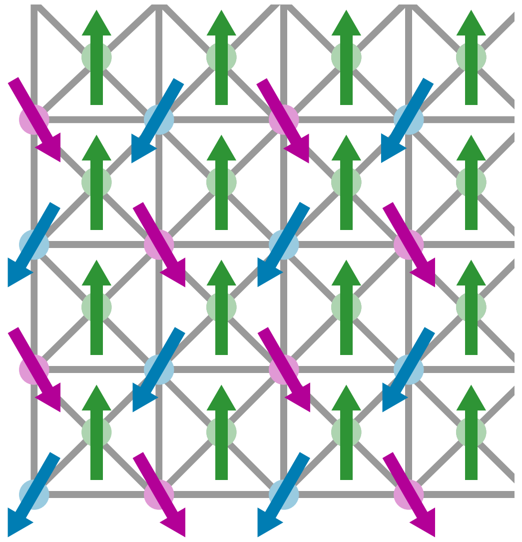


C decoupled

(π,π) Néel AB

$m^{A}_{Fe} = m^{B}_{Fe} = m^{AB}_{AF}$

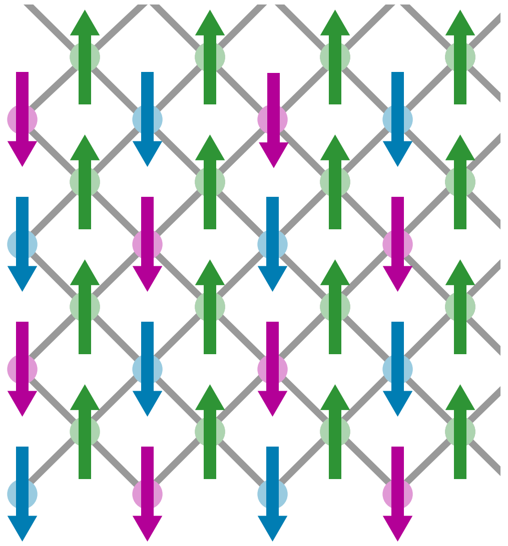
α = 0.5 (J'=J)



(120º) Néel ABC

$m^{A}_{Fe} = m^{B}_{Fe} \neq m^{AB}_{AF}$

α = 1.0 (J=0)



(π,π) Néel ABC

$m^{C}_{Fe} = m^{AB}_{Fe} = m^{ABC}_{AF}$

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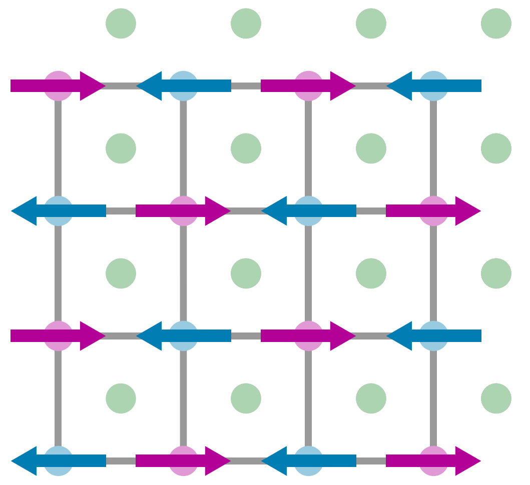

C decoupled
(π,π) Néel AB

$m_{Fe}^{A} = m_{Fe}^{B} = m_{AF}^{AB}$

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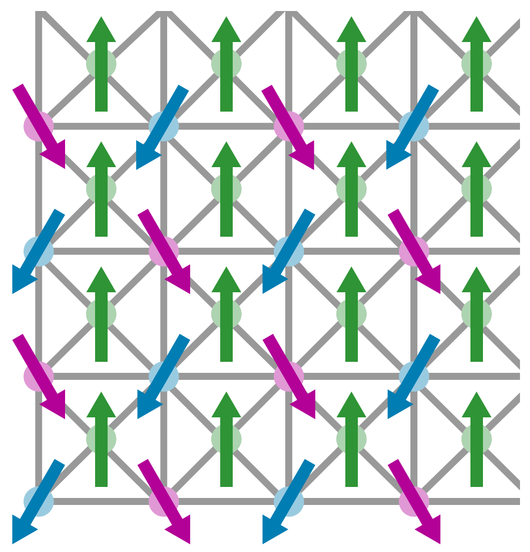

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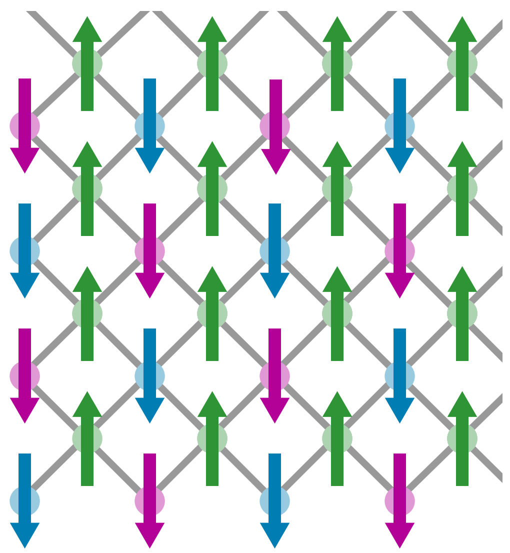
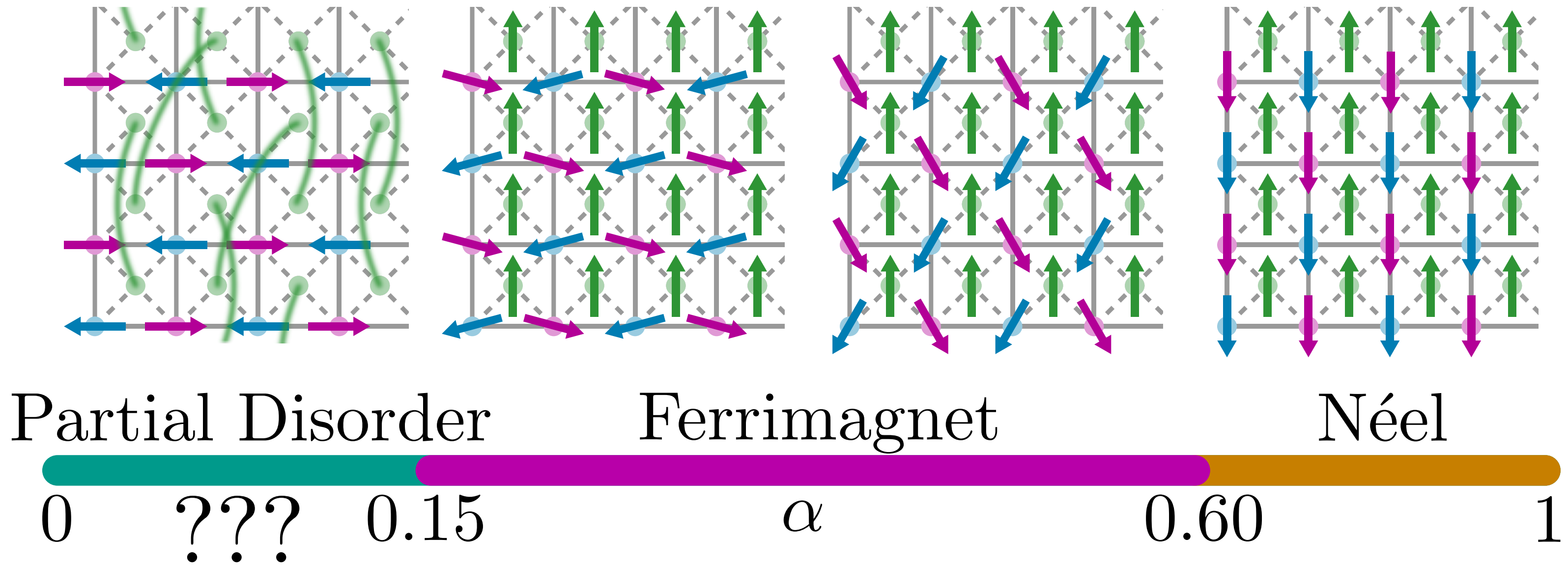


(π,π) Néel ABC

$m_{Fe}^{C} = m_{Fe}^{AB} = m_{AF}^{ABC}$

Phase Diagram:



Sublattice magnetizations

I) Partial Disorder?

$$m^{A}_{Fe} = m^{B}_{Fe} = m^{AB}_{AF}$$

II) Ferrimagnet:

$$m^{A}_{Fe} = m^{B}_{Fe} \neq m^{AB}_{AF}$$

III) (π,π) Néel ABC:

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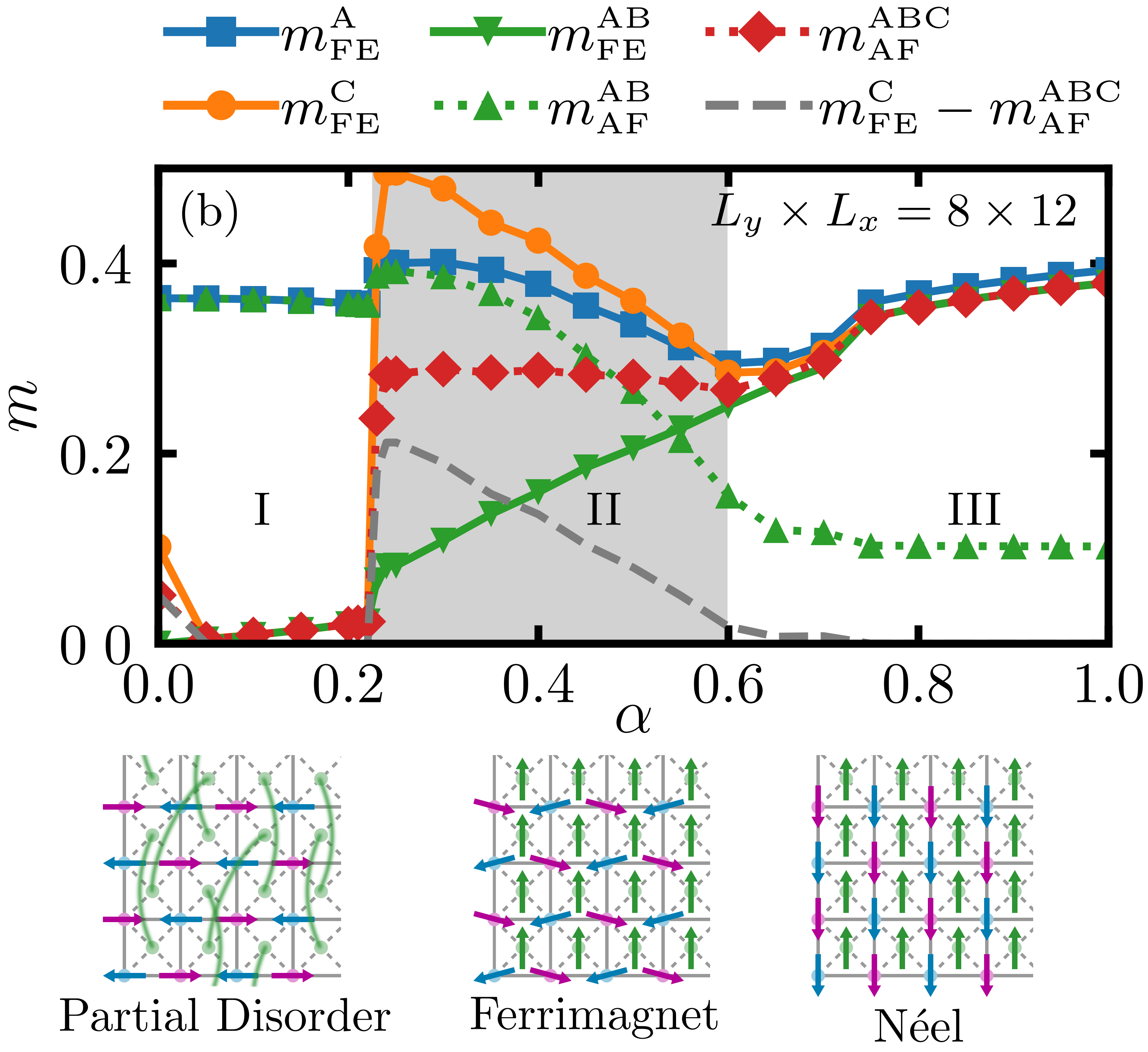
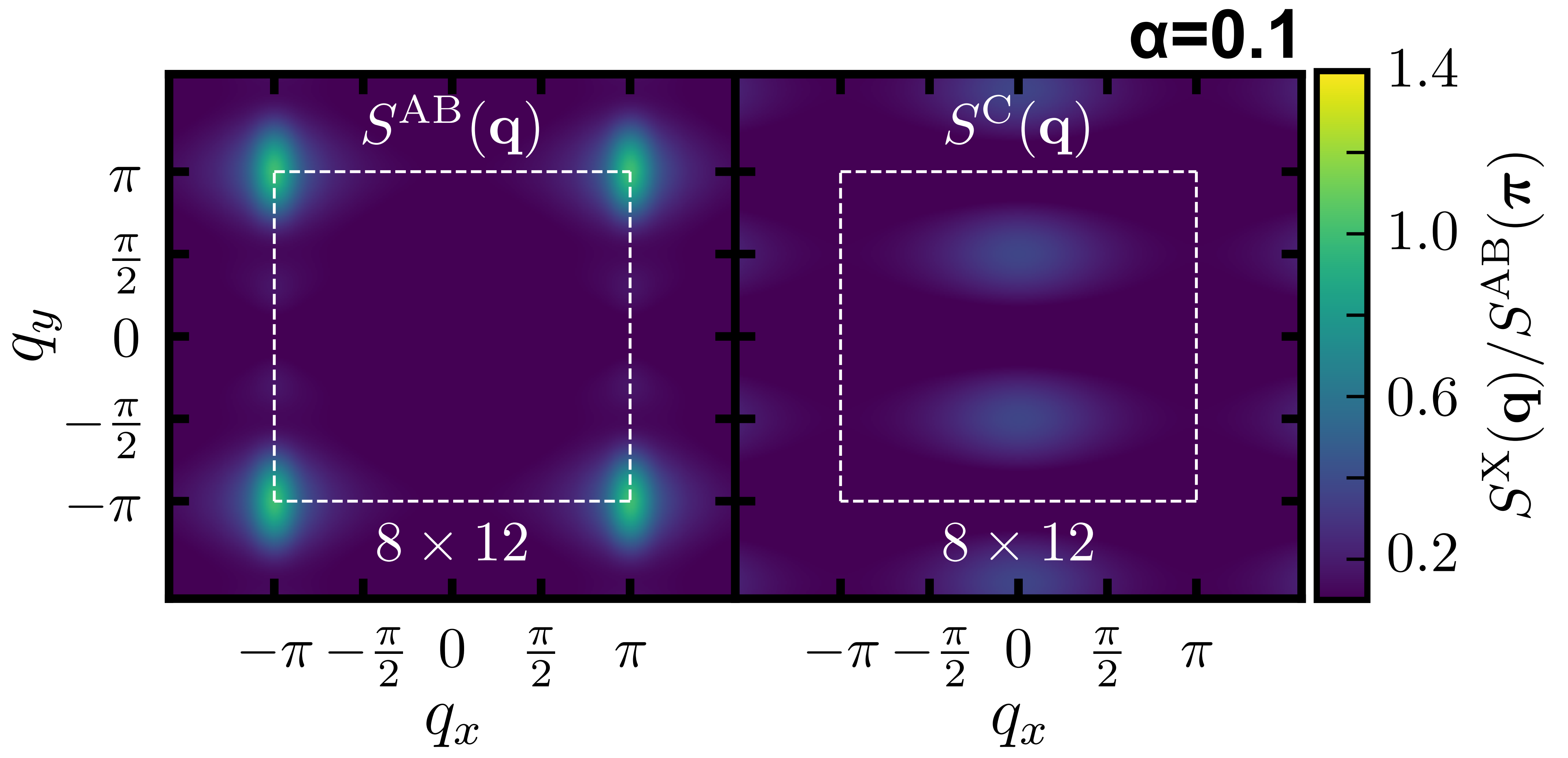


Magnetic structure factor

Peaks
- Néel order in AB sublattice
- Signal in the C sublattice

    Spiral π/2 or stripes <2>

<2> = 



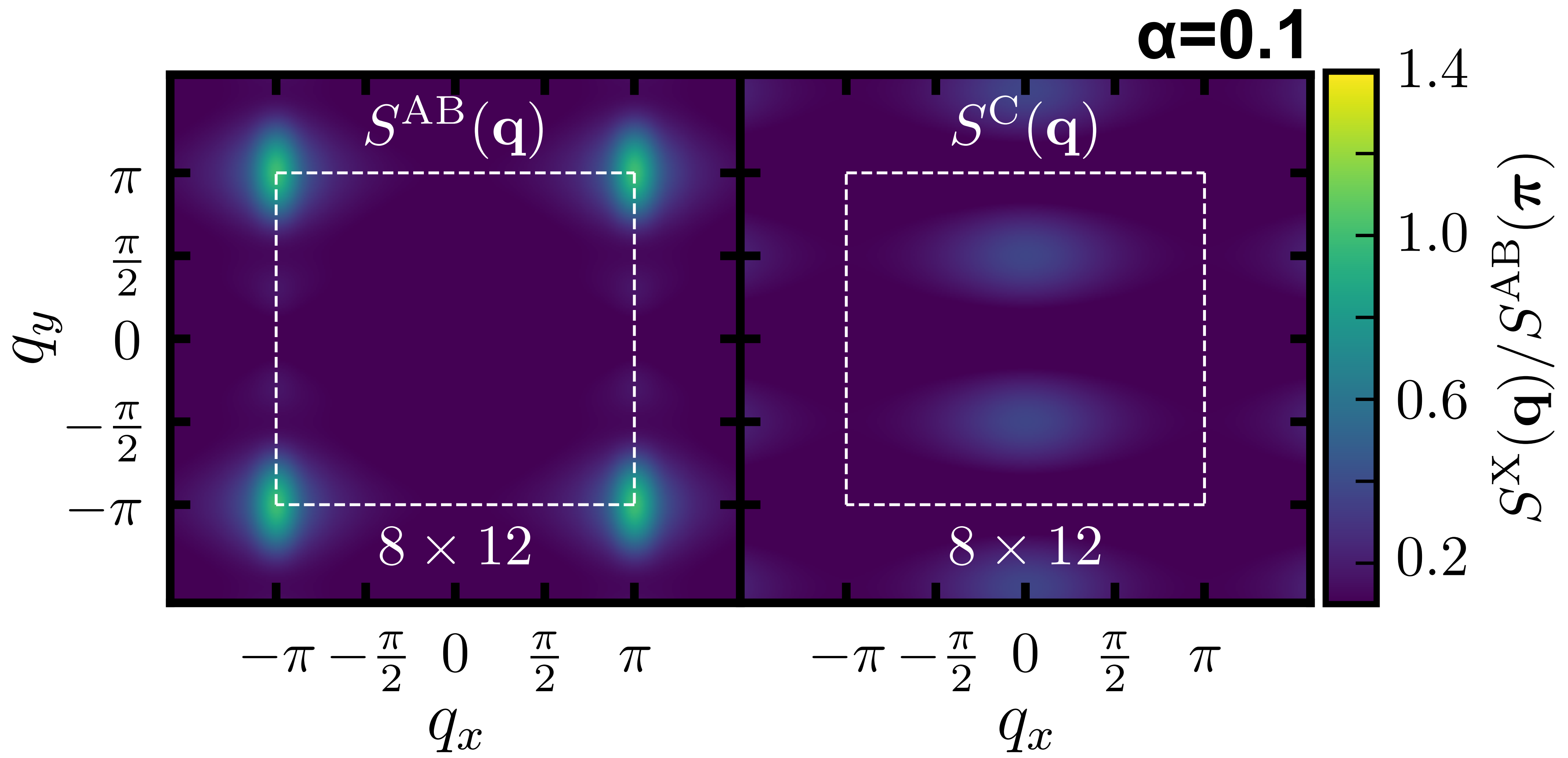
Magnetic structure factor

Peaks

- Néel order in AB sublattice
- Signal in the C sublattice

  Spiral π/2 or stripes <2>

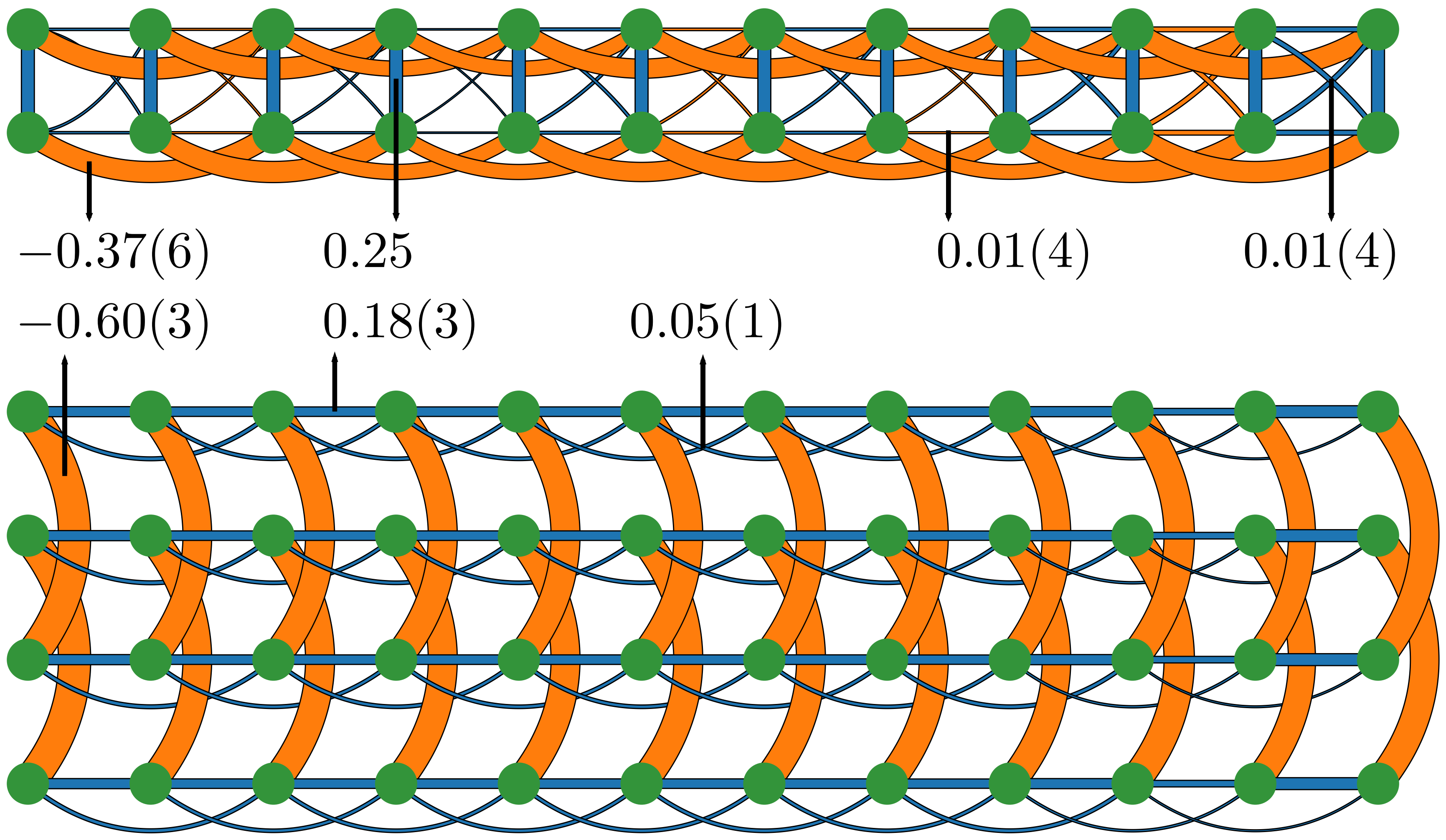
  <2> = ↑↑↓↓↑↑ ooo

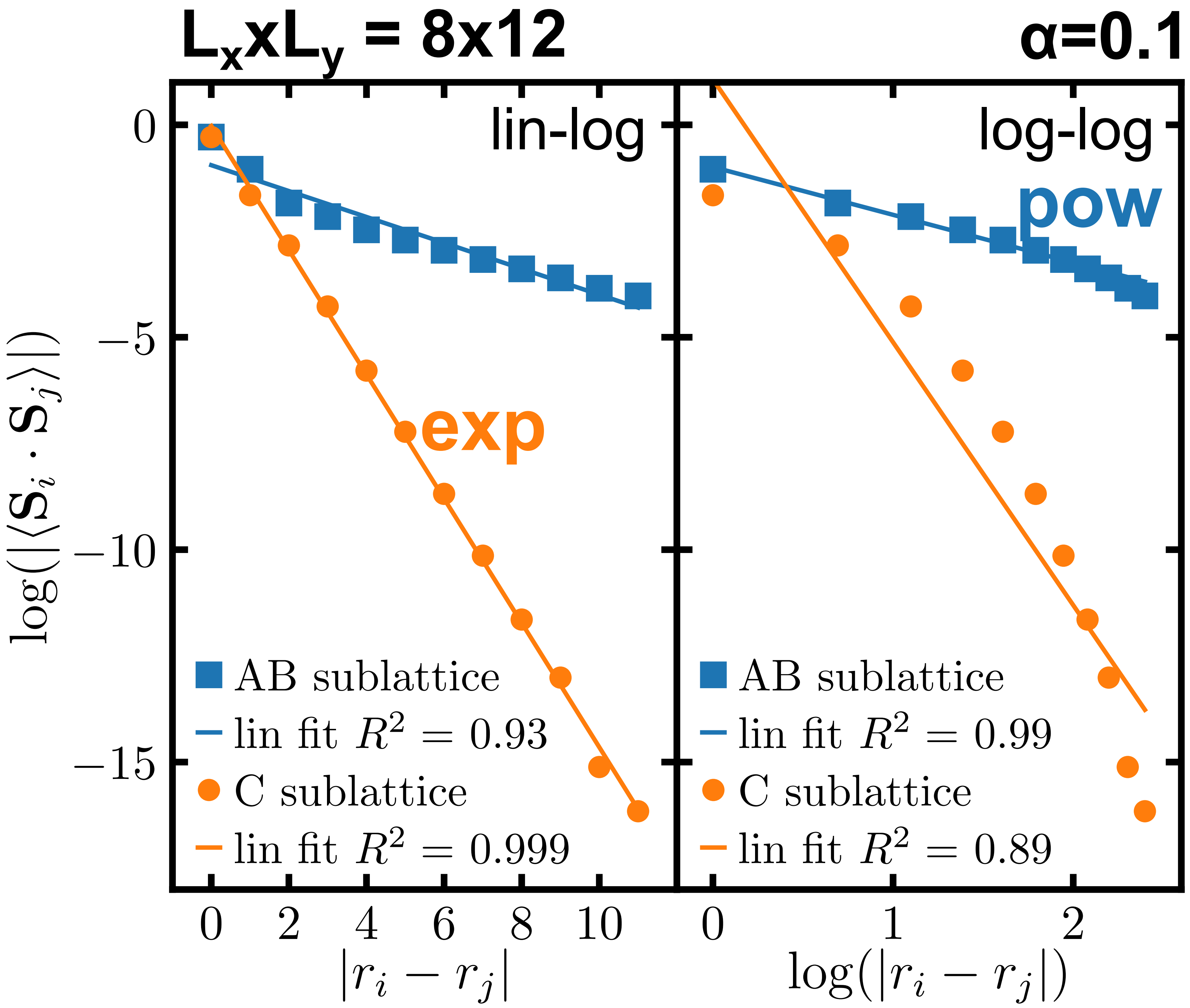


Finite size behaviour

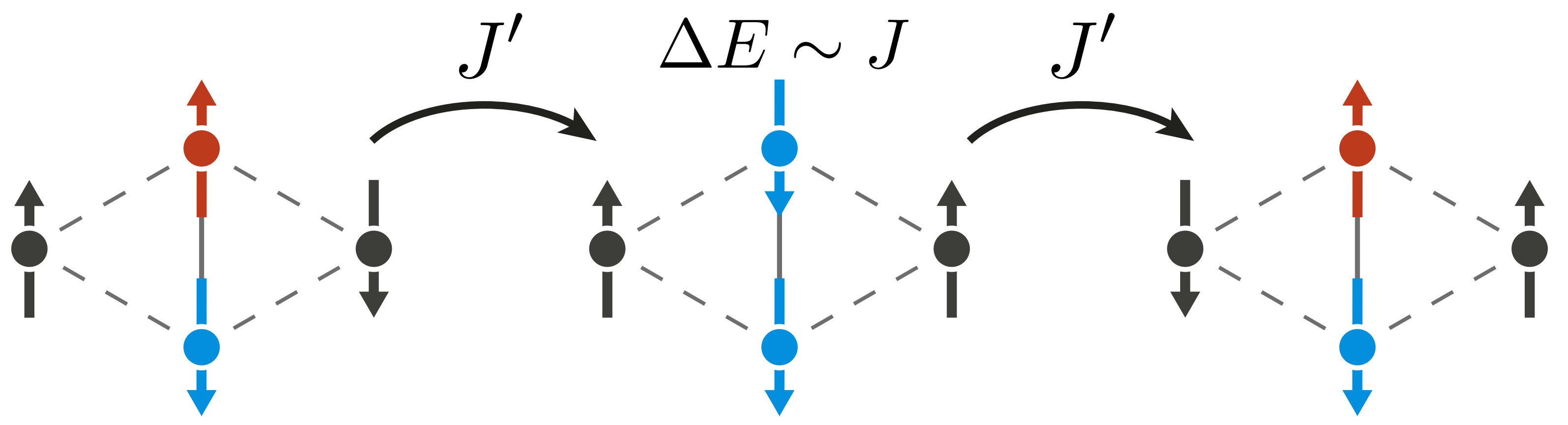
- Néel AB peaks grow
- Stripes C peaks shrink

Real space correlations

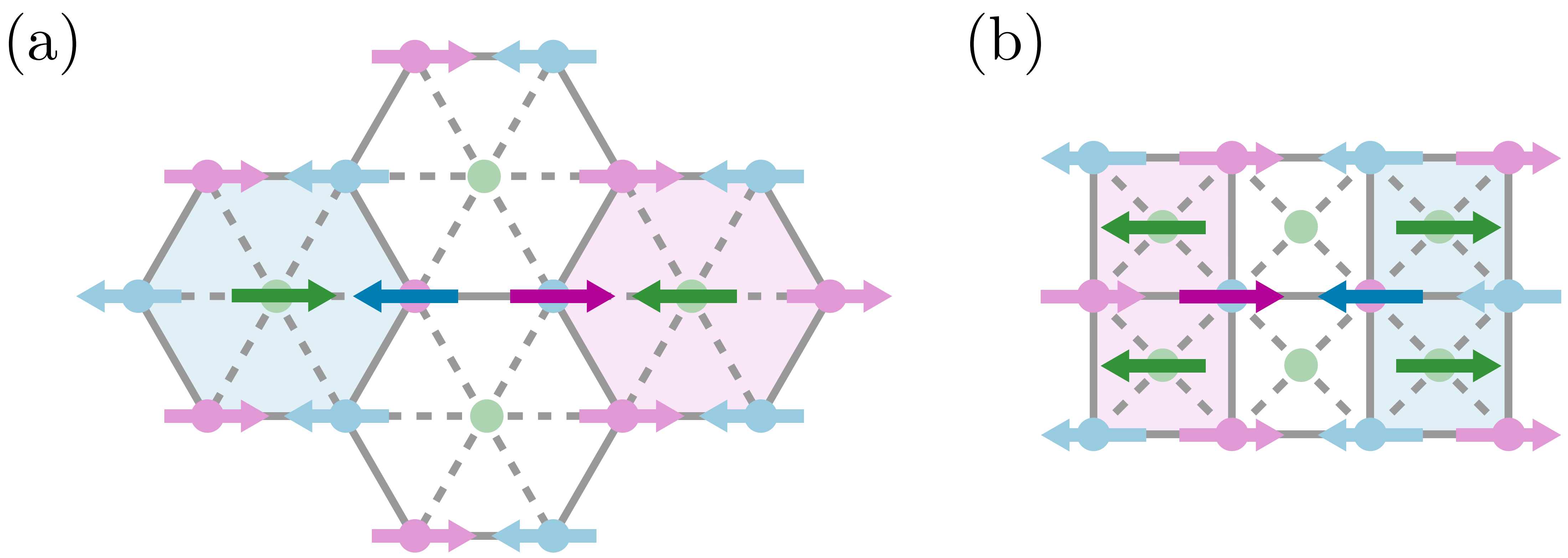




Hand-wavy arguments for an C-spins effective model



- 1st and 2nd neighbours: xy-Ferro



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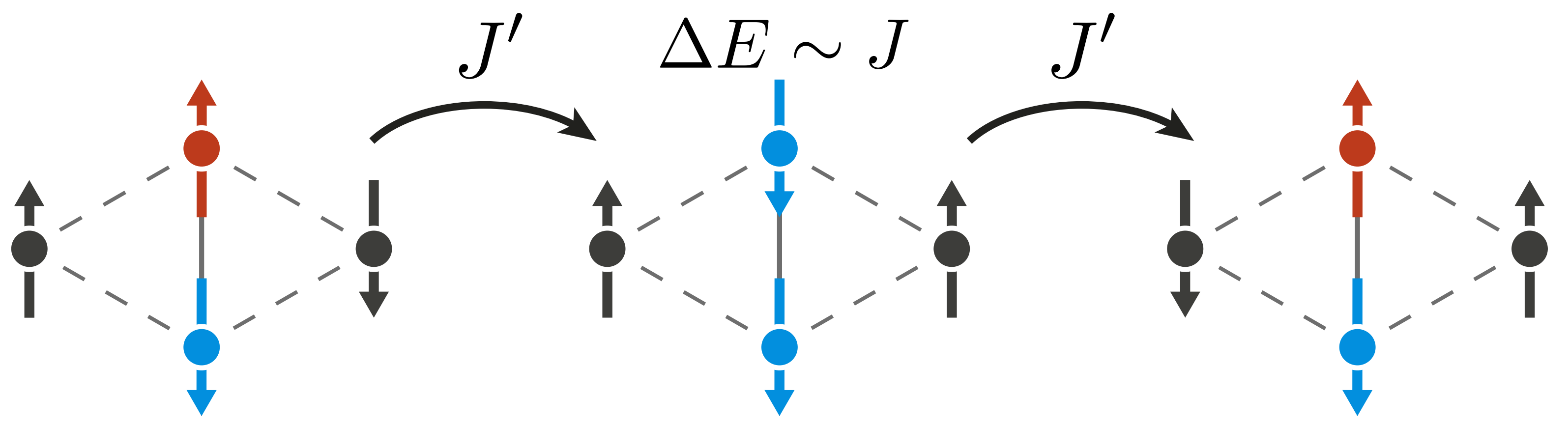
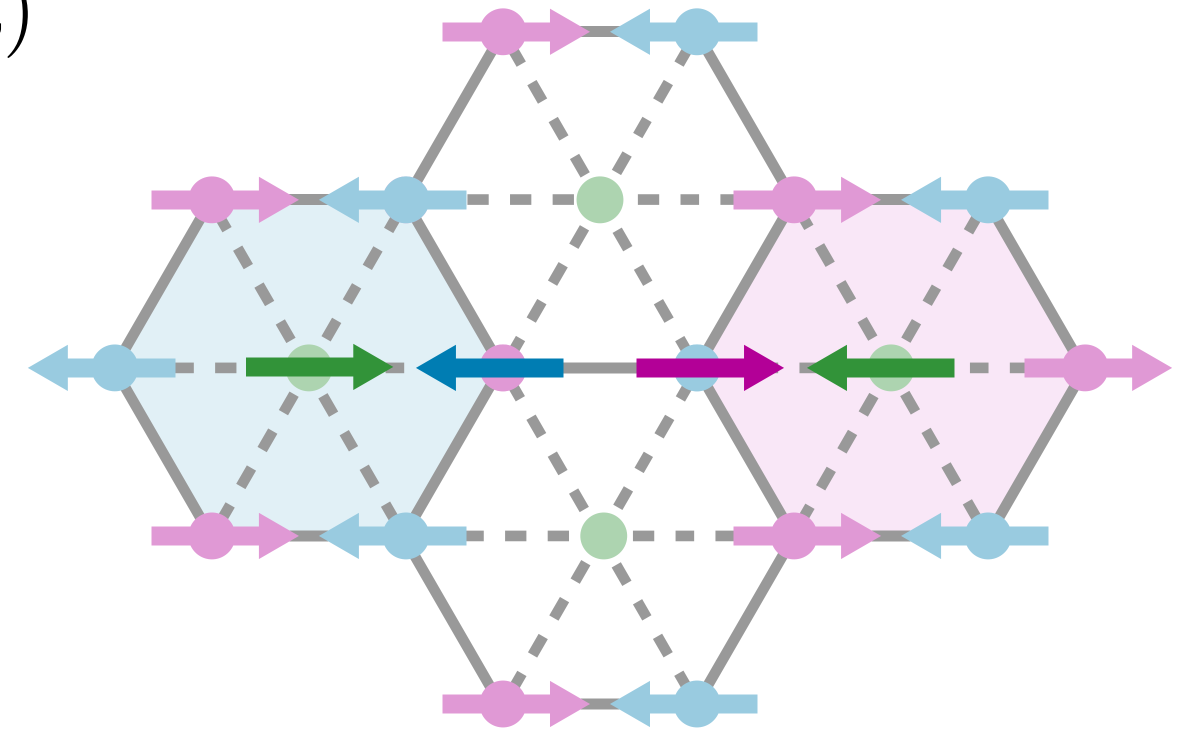
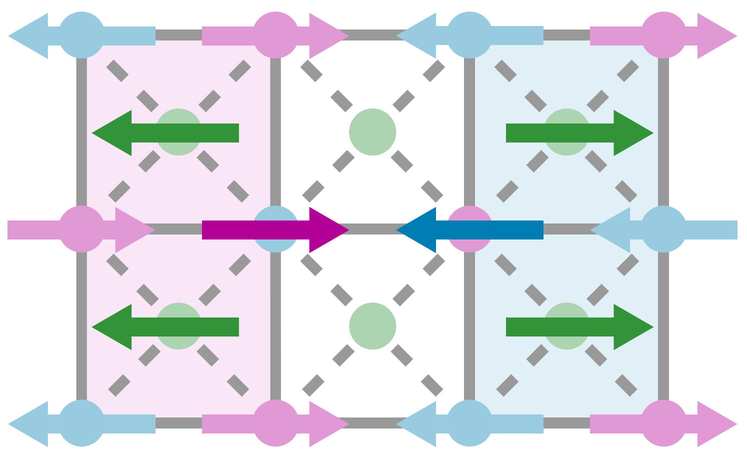


- 1st and 2nd neighbours: xy-Ferro
- 3rd and 4th neighbours: z-AF

(a)

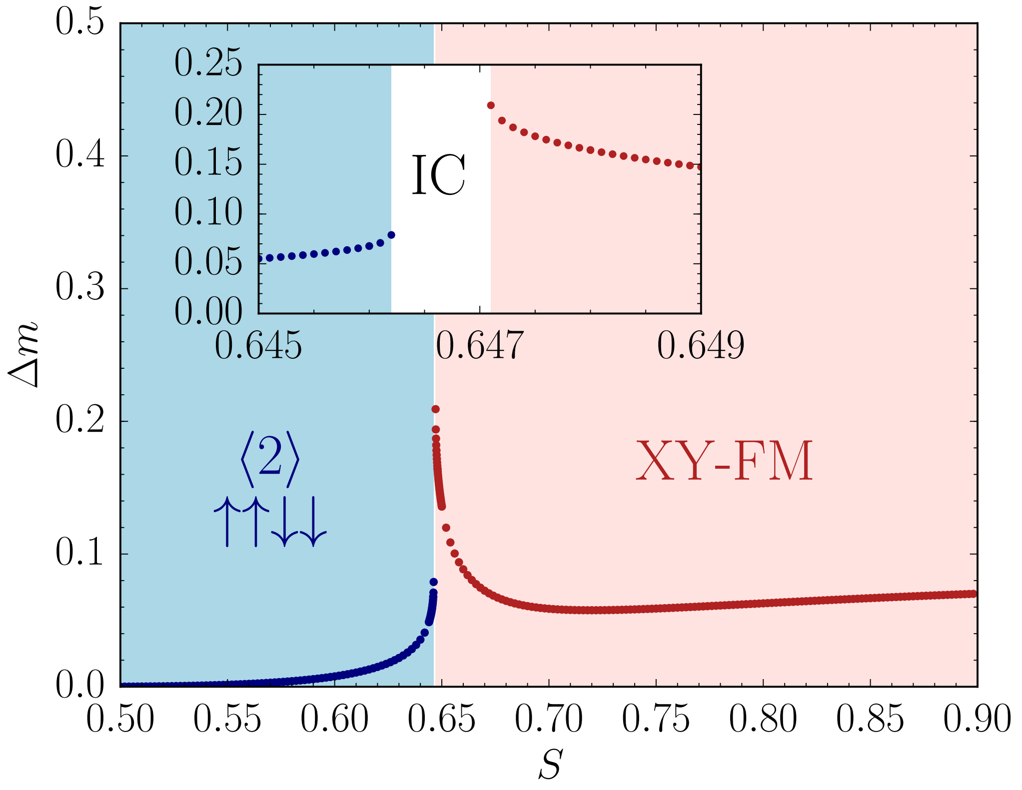


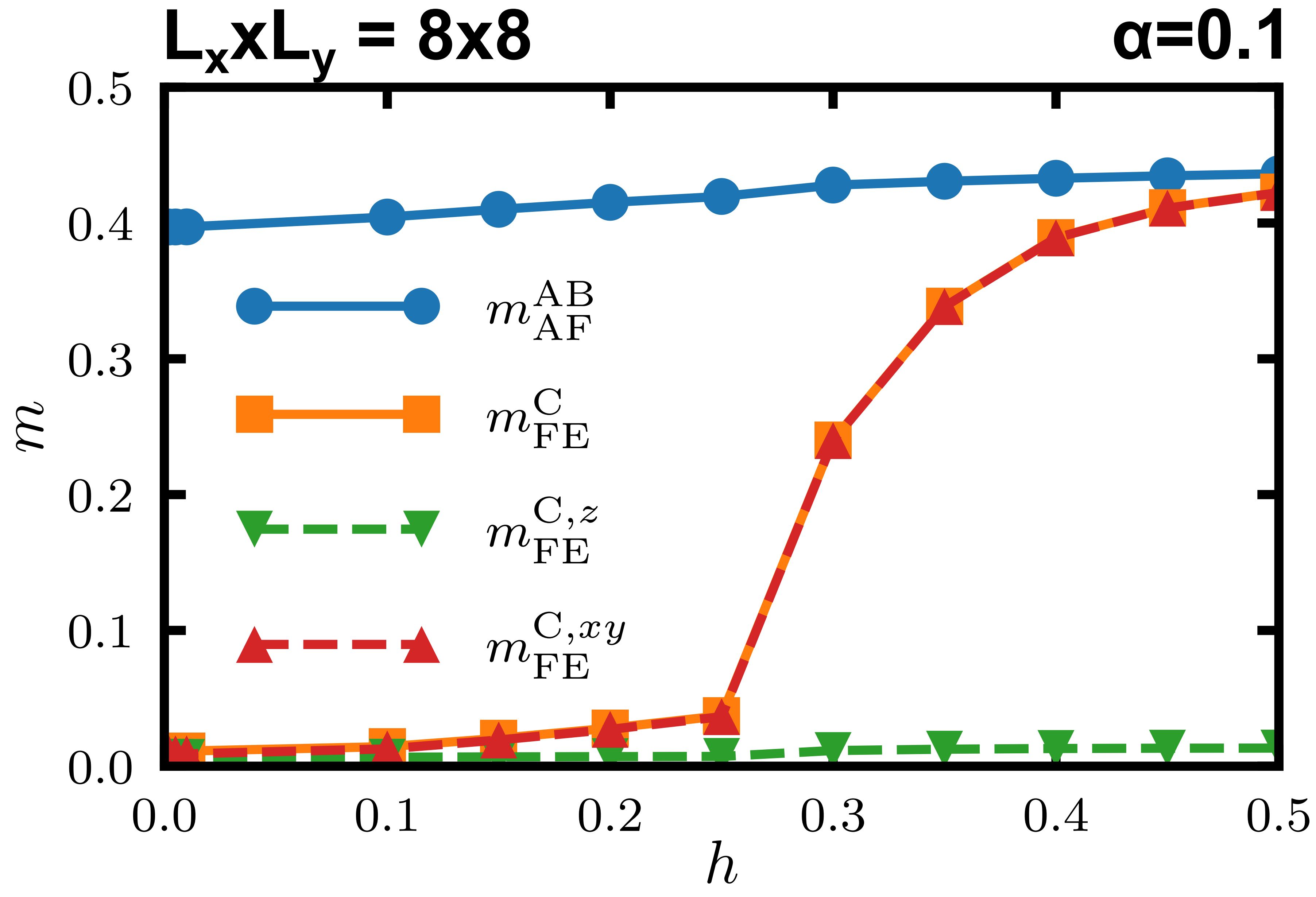
(b)



<2> compatible

Reduced quantum fluctuations





- Large-S → reduced fluctuations
- Mag field along z? (borders)
- It reduces fluctuations in AB
- C correlations are Fe in-plane (xy)

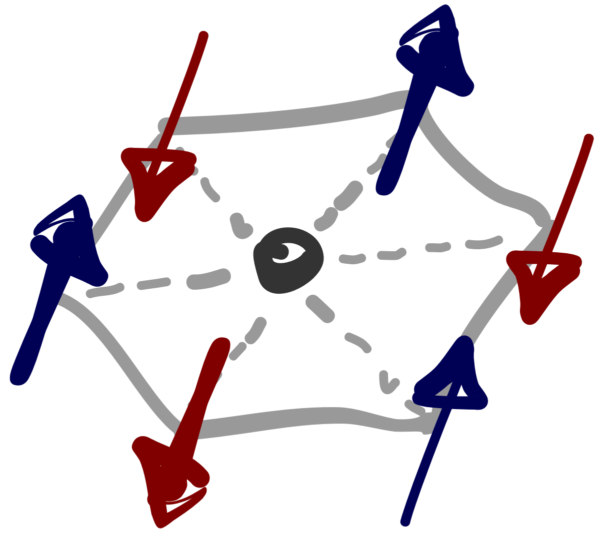
Conclusions

- Partial disorder at T=0
- Correlated C sites due to ZPQF
- Consistent with the effective model
- Tunable C correlations via AB

PD in isotropic Heisenberg antiferromagnets at T=0

Studied models

- Stuffed honeycomb lattice
  Gonzalez, FL, Blesio, Trumper, Gazza and Manuel, PRL 122, 017201 (2019)



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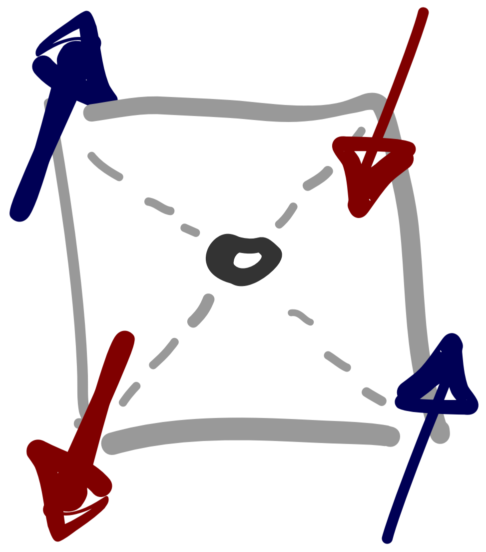
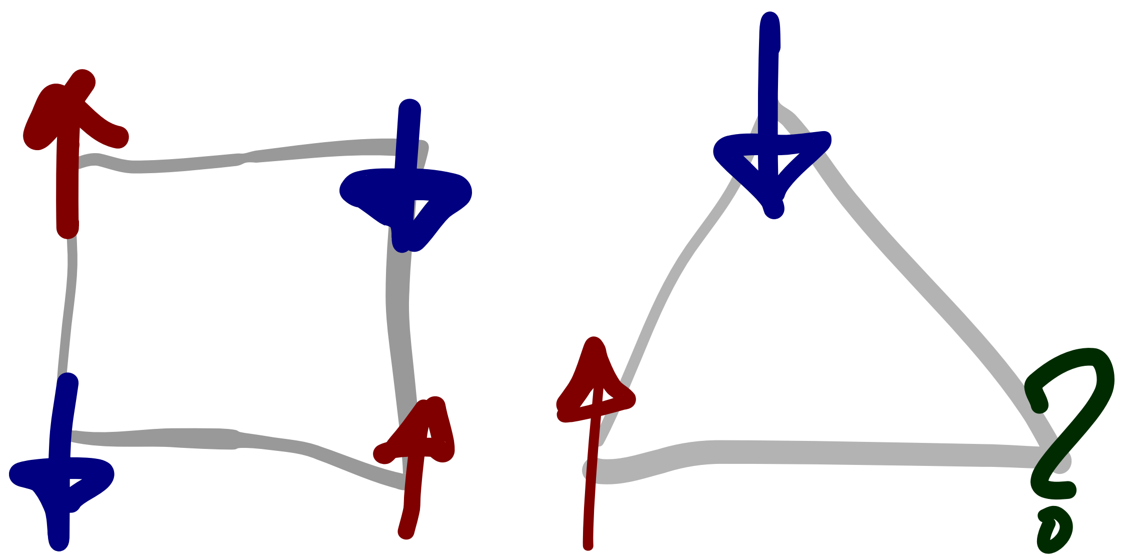
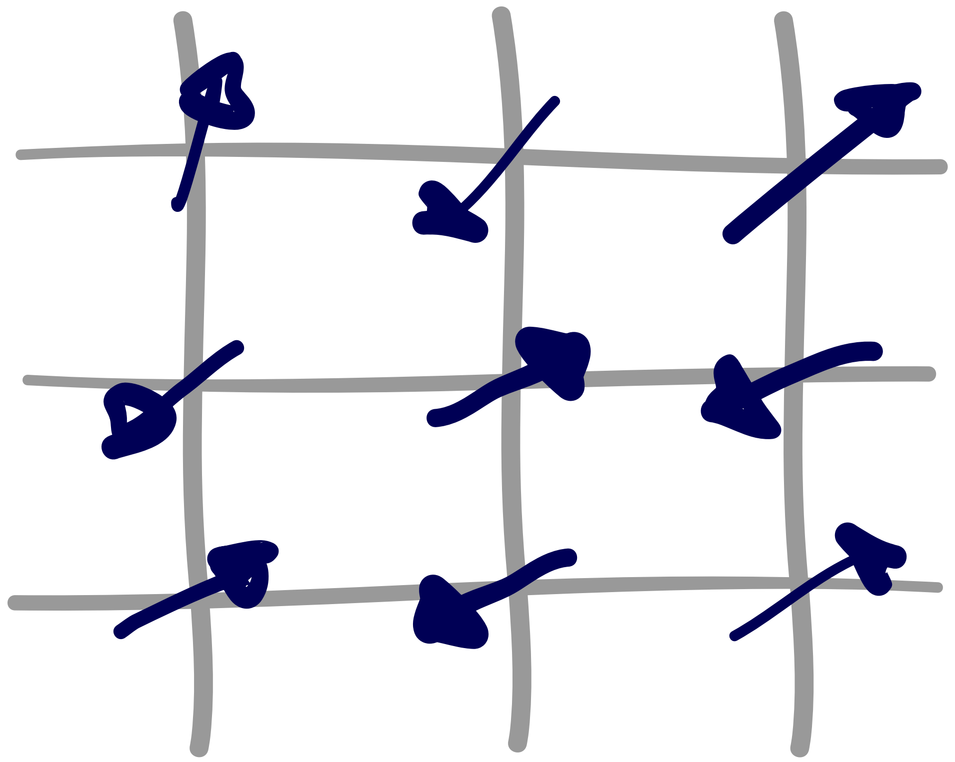
  Blesio, FL and Gonzalez, PRB 107, 134418 (2023)



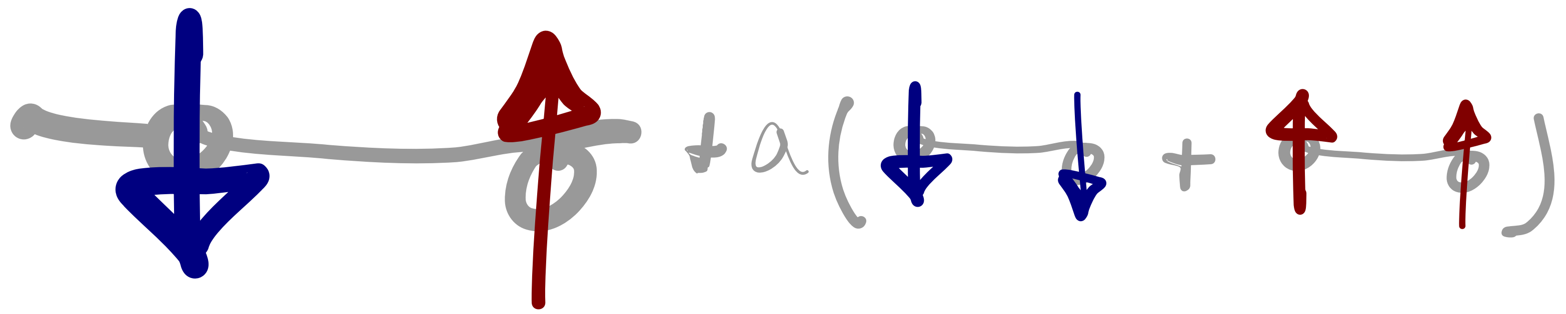
Two-dimensional spin systems

Magnetic Frustration





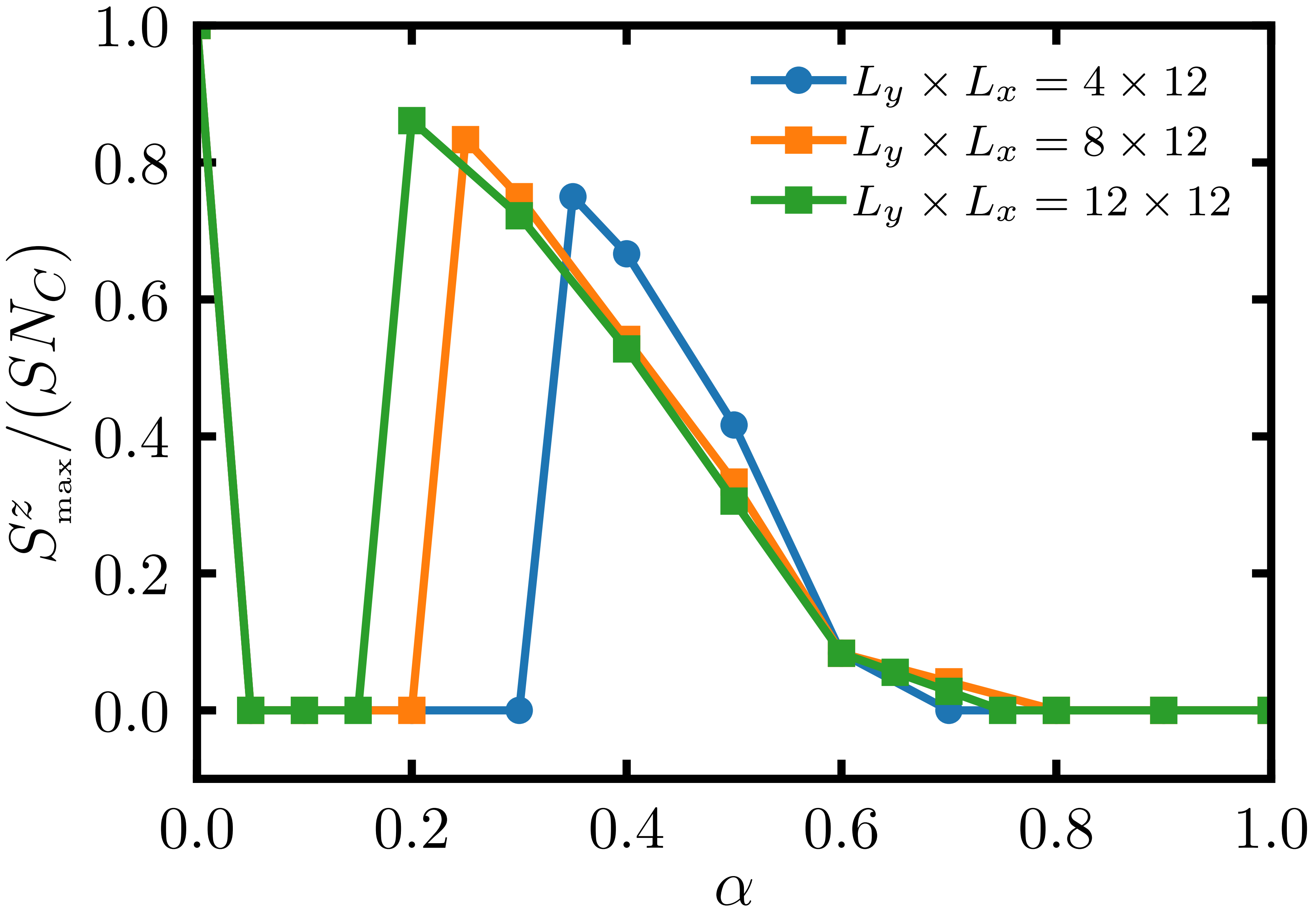
Zero-point quantum fluctuations

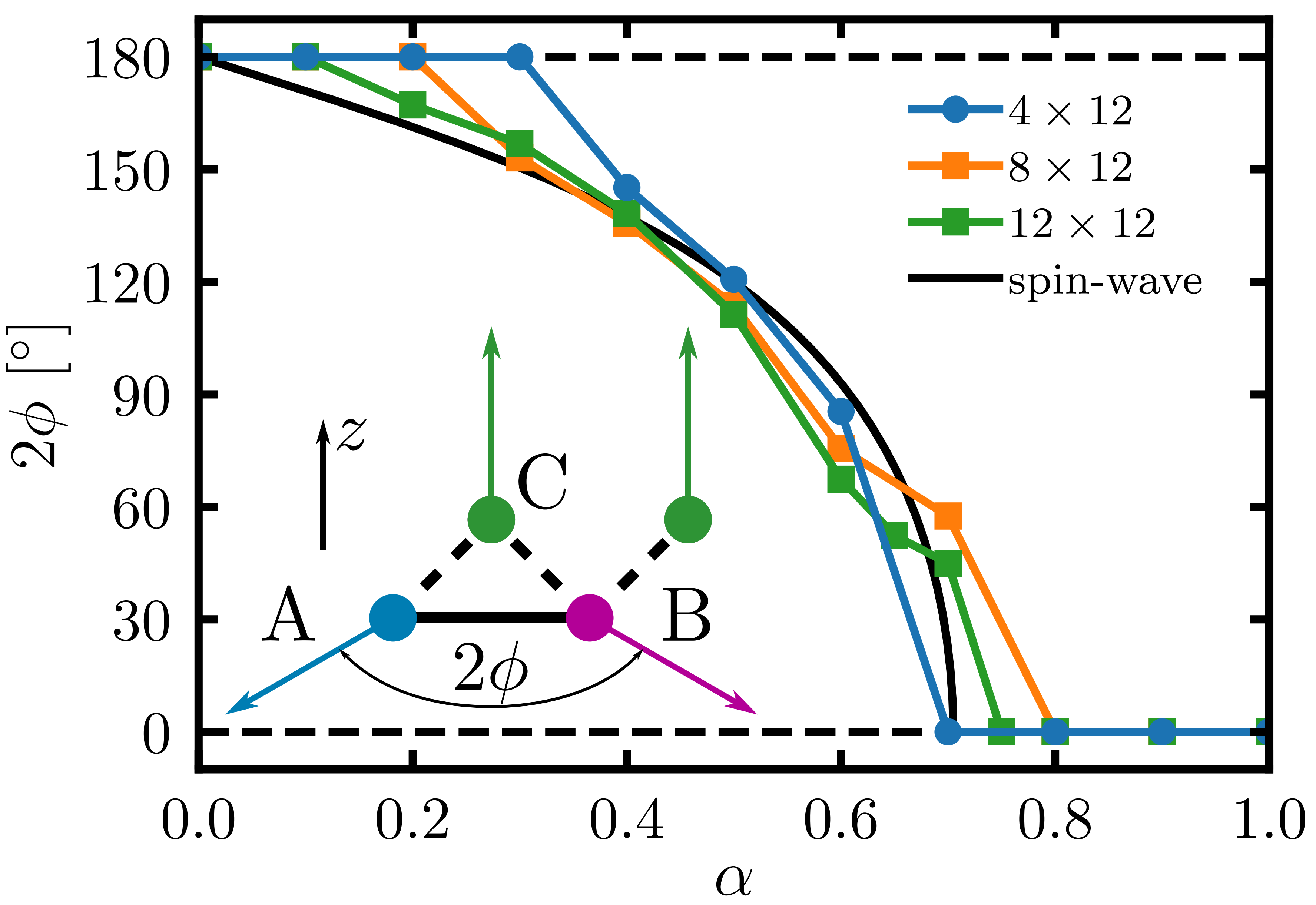


Collective phenomena

- Spin liquids
- Order by disorder
- Partial disorder

EXTRA: S^z_{max} and angle





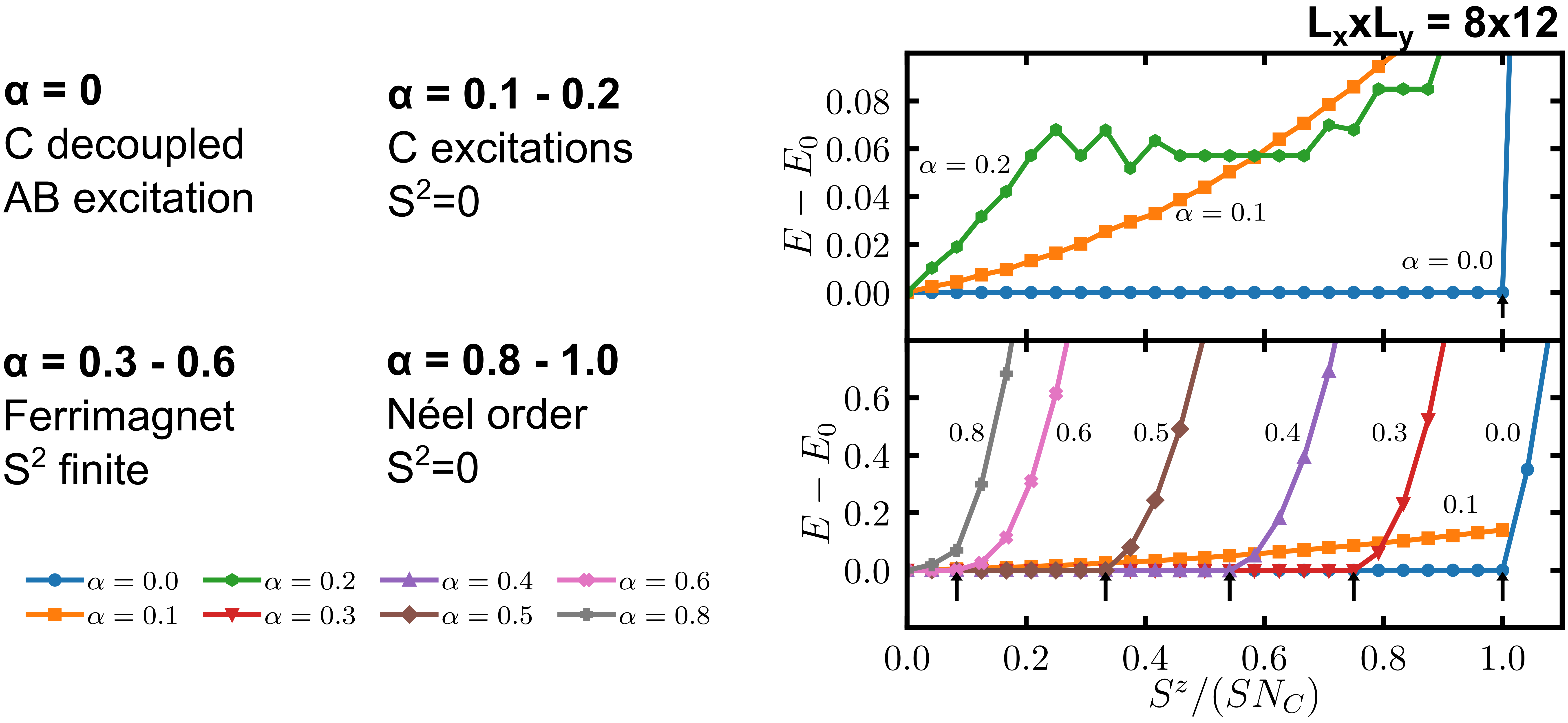
Energy difference vs S^z sector

α = 0
C decoupled
AB excitation

α = 0.1 - 0.2
C excitations
S^2=0

α = 0.3 - 0.6
Ferrimagnet
S^2 finite

α = 0.8 - 1.0
Néel order
S^2=0



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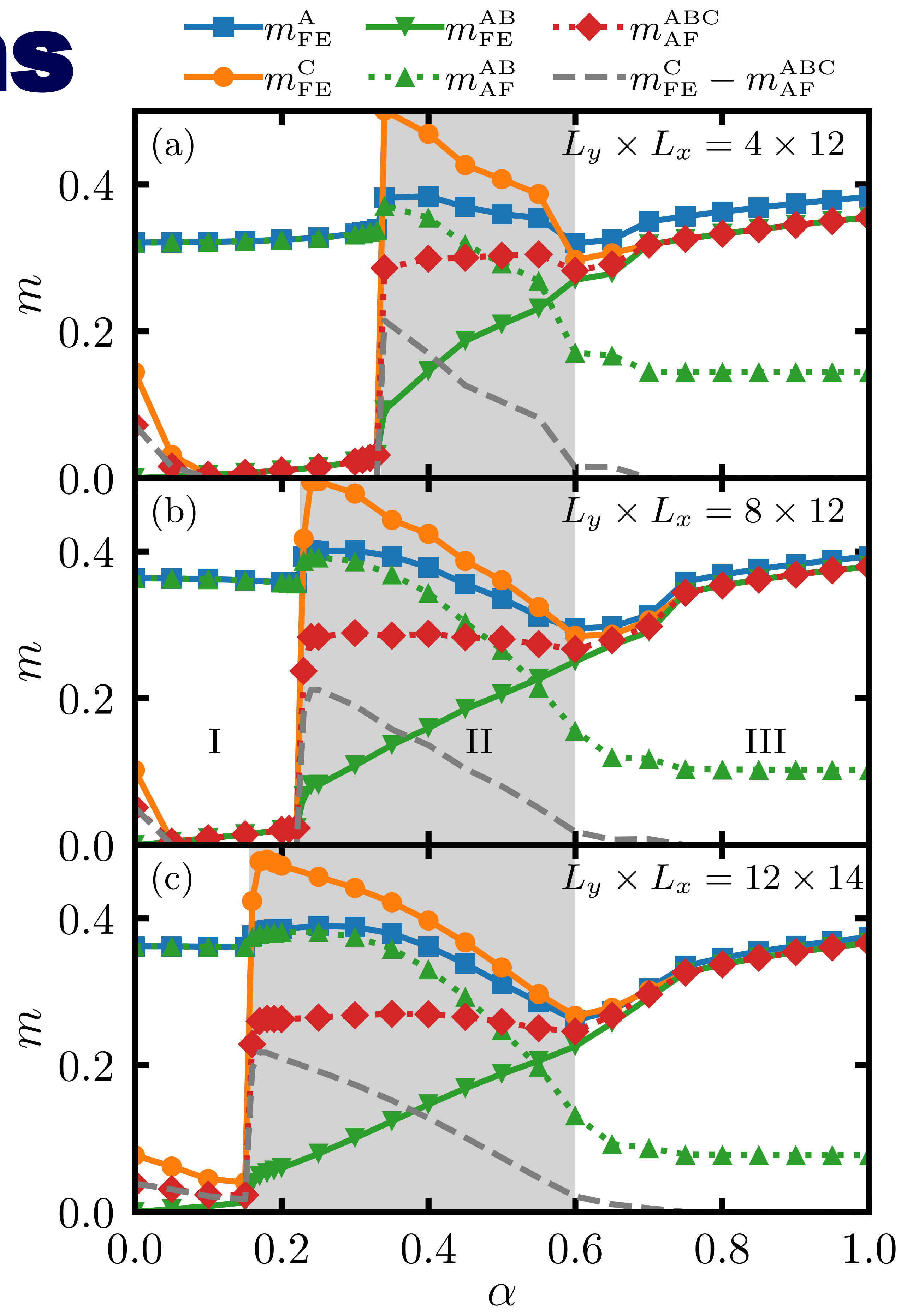

Magnetic structure factor

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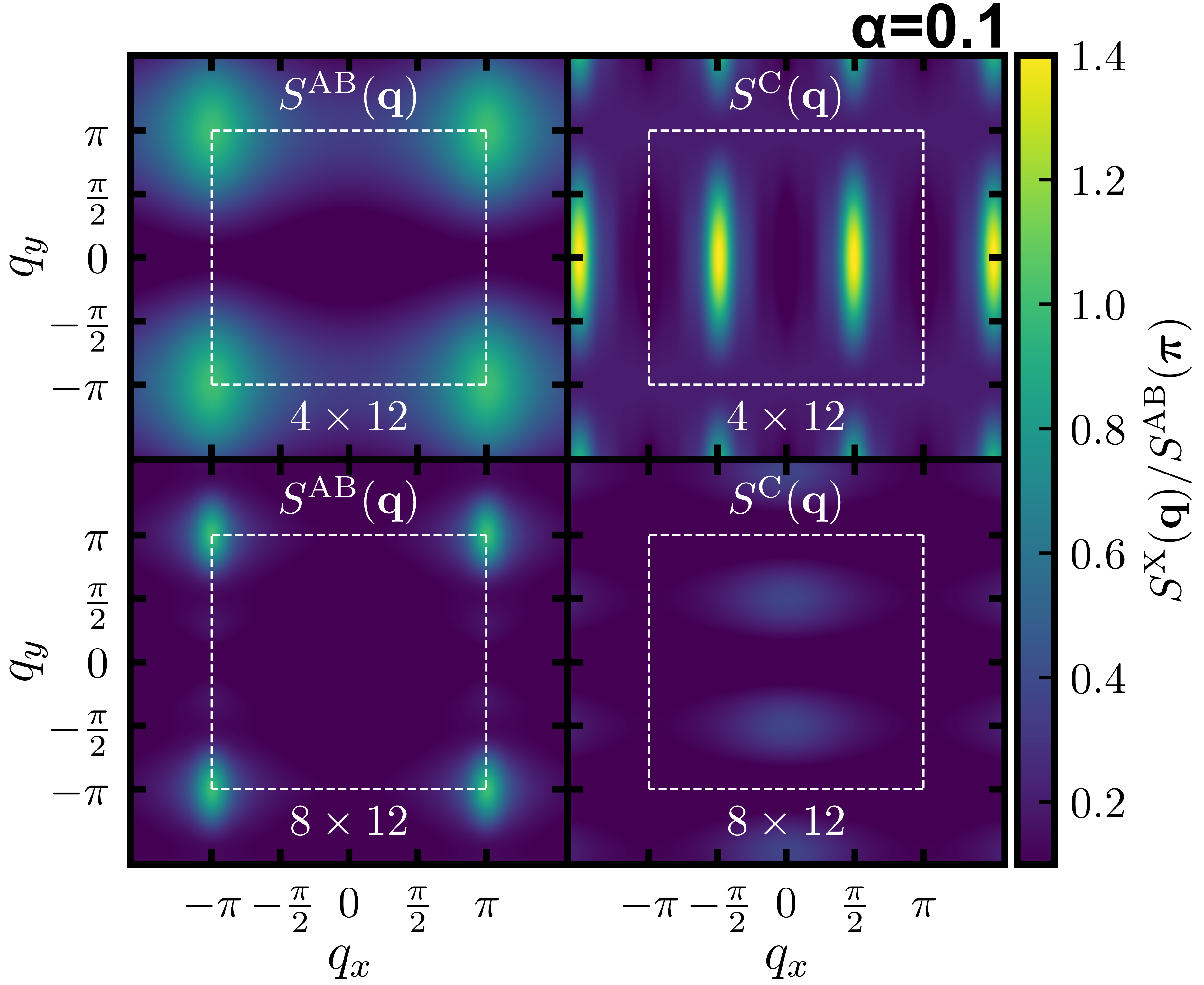

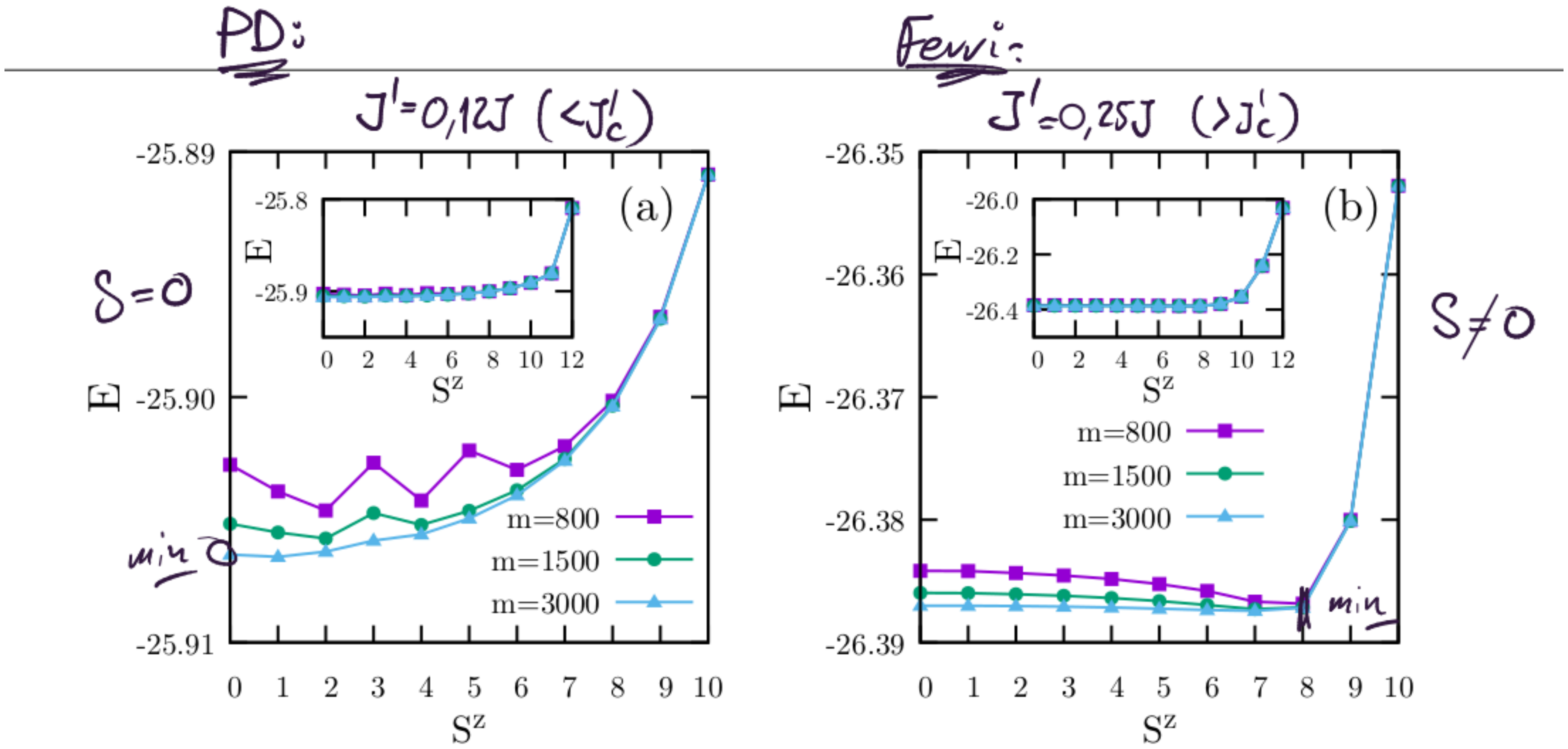

Fig. 3.9: Cálculo de la energía dentro de cada subespacio S_z con DMRG. En el panel izquierdo, un singlete para la fase parcialmente desordenada $J' = 0.12 < J'_c$. En el panel derecho, un ferrimagneto para la fase con orden semiclásico $J' = 0.25 > J'_c$.

"Almost Lx-independent"